1 Back to fundamentals [Warmup: 10 points]

Consider the two-player game described by the payoff matrix in Figure 1. The rows and columns correspond to player A’s and player B’s actions respectively. Each cell contains player A’s payoff, followed by player B’s payoff. Determine all the Nash equilibria (pure and mixed). Find the dominant strategies, if any.

Hint: In a mixed Nash equilibrium, a player never assigns non-zero probability to a strictly dominated strategy.

2 Mafia internal affairs [20 points]

Consider a Mafia clan with 4 members. There is a strict hierarchy between the members, as shown in Figure 2. The clan has just made $10000 and the time has come to divide up the loot between the members.

The rules for loot redistribution are the following. First, the highest member in the hierarchy proposes an allocation and this allocation is put to vote. If the allocation is approved (by majority or tie), it gets implemented. Else, the highest ranking member (who has now lost the approval of the clan) is killed and the next highest member of the hierarchy gets to propose an allocation. This process continues until an allocation gets approved.

Each clan member is assumed to vote with the following considerations in decreasing order of priority: (i) stay alive, (ii) make as much money as possible, (iii) money remaining constant, kill off as many higher ranking members as possible. You may assume that the smallest unit of money is $1.

What allocation should the boss propose? Interpret this allocation. Does it agree with your intuition?

3 Dealing with the Mafia [20 points]

You are a poor businessman. You have $1 and you owe the Mafia $5. Your only choice for staying alive is to make a series of bets with a fair coin. When you bet $r$ dollars, you win $r$ dollars if the coin flips in your favor (this happens with probability 1/2); else you lose $r$ dollars. You have the choice between two strategies. Bet $1 each time (the timid strategy), or bet as much as you can but no more than necessary to reach a target fortune of $5 (the bold strategy). If you succeed in making $5, your life is spared; if you go bankrupt, the mafia will execute you. Find your probability of staying alive with each strategy.
4 Alternating offers [20 points]

Adam has made a deal with Amanda and Yu regarding the grading of HW5. If they manage to finish grading all HW5 before the deadline, they will receive a bonus of $5 in total. Of course, the need to cooperate to complete the grading, so they need to figure out how to divide the bonus if they’re successful.

To do this, the agree on the following scheme. Yu first makes an offer to Amanda. His offer could be any number of \{0, 1, 2, 3, 4, 5\}. Amanda can then either accept the offer or decline it. If Amanda accepts the offer, she will get that amount of money and Yu will take the rest. If Amanda declines the offer, then Amanda can counter-offer to Yu following the same rules. In order to save enough time for grading, they agree to stop after \(N\) offers have been made in total. If these \(N\) offers are all rejected, they will not grade HW5 and thus receive no bonus.

To given you a feel for this game, consider the case of \(N = 1\). Here, only Yu can make an offer. In this case, the following is an equilibrium. Amanda only accepts $4, $5 and rejects $0, $1, $2, $3. Yu offers $4. This is an equilibrium profile because neither Amanda nor Yu can possibly earn more if one of them is allowed to change the strategy while the other’s strategy is fixed.

Your task:

(a) If \(N = 88\), find the best strategy for Amanda.

(b) If \(N = 99\), find the best strategy for Amanda.