CS137: Electronic Design Automation

Day 20: November 23, 2005
Scheduling
Variants and Approaches

Today

- Scheduling
  - Force-Directed
  - SAT/ILP
  - Branch-and-Bound

Last Time

- Resources aren’t free
- Share to reduce costs
- Schedule operations on resources
- Greedy approximation algorithm

Force-Directed

- Problem: how exploit schedule freedom (slack) to minimize instantaneous resources
  - Directly solve time constrained
  - Trying to minimize resources

Force-Directed

- Given a node, can schedule anywhere between ASAP and ALAP schedule time
  - Between latest schedule predecessor and ALAP
  - Between ASAP and already scheduled successors
- N.b.: Scheduling node will limit freedom of nodes in path

Single Resource Challenge
**Force-Directed**

- If everything were scheduled, except for the target node, we would:
  - examine resource usage in all timeslots allowed by precedence
  - place in timeslot which has least increase in maximum resources

**Force-Directed**

- **Problem:** don’t know resource utilization during scheduling
- **Strategy:** estimate resource utilization

**Force-Directed Estimate**

- Assume a node is uniformly distributed within slack region:
  - between earliest and latest possible schedule time
- Use this estimate to identify most used timeslots

**Single Resource Challenge**
Force-Directed

- Scheduling a node will shift distribution
  - all of scheduled node’s cost goes into one timeslot
  - predecessor/successors may have freedom limited so shift their contributions
- Want to shift distribution to minimize maximum resource utilization (estimate)
Single Resource Challenge

Single Resource Challenge

Single Resource Challenge

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Single Resource Challenge
**Force-Directed Algorithm**

1. ASAP/ALAP schedule to determine range of times for each node
2. Compute estimated resource usage
3. Pick most constrained node (in largest time slot…)
   - Evaluate effects of placing in feasible time slots (compute forces)
   - Place in minimum cost slot and update estimates
   - Repeat until done

**Time**

- Evaluate force of putting in timeslot $O(NT)$
  - Potentially perturbing slack on net prefix/postfix for this node $\rightarrow N$
  - Each node potentially in $T$ slots
- Evaluate all timeslots can put in $O(NT^2)$
- $N$ nodes to place
- $O(N^2T^2)$
  - Loose bound—don’t get both $T$ slots and $N$ perturbations
SAT/ILP (Integer-Linear Programming)

Two Constraint Challenge
- Processing elements have limited memory
  - Instruction memory (data memory)
- Tasks have different requirements for compute and instruction memory
  - \( i.e. \) Run length not correlated to code length

Task
- **Task**: schedule tasks onto PEs obeying both memory and compute capacity limits

Example and ILP solution
From Plishker et al. NSCD2004

Task
- **Task**: schedule tasks onto PEs obeying both memory and compute capacities
- \( \rightarrow \) two capacity partitioning problem
  - \( \ldots \) actually, didn’t say anything about communication…
- \( \rightarrow \) two capacity bin packing problem
- Task: \( i <C_i, I_i> \)

SAT Packing
- \( A_{ij} \) – task \( i \) assigned to resource \( j \)

Constraints
- Coverage constraints
- Uniqueness constraints
- Cardinality constraints
  - PE compute
  - PE memory

Allow Code Sharing
- Two tasks of same type can share code
- Instead of memory capacity
  - Vector of memory usage
- Compute PE Imem vector
  - As OR of task vectors assigned to it
- Compute mem space as sum of non-zero vector entries
Allow Code Sharing

- Two tasks of same type can share code
- Task has vector of memory usage
  - Task $i$ needs set of instructions $k$: $T_{i,k}$
- Compute PE Imem vector
  - OR (all $i$): $PE\text{.Imem}_{j,k} = A_{i,j} \cdot T_{i,k}$
- PE Mem space
  - $PE\text{.Total}_\text{Imem}_{j} = \sum (PE\text{.Imem}_{j,k} \cdot \text{Instrs}(k))$

Symmetries

- As with partitioning, many symmetries
- Speedup with symmetry breaking
  - Tasks in same class are equivalent
  - PEs indistinguishable
  - Total ordering on tasks and PEs
  - Add constraints to force tasks to be assigned to PEs by ordering
  - Plishker claims “significant runtime speedup”
  - Using GALENA [DAC 2003] pseudo-Boolean SAT solver

Plishker Task Example

Results

- Greedy (first-fit) binpack
- SAT/ILP Solve

Why can they do this?

- Ignore precedence?
- Ignore Interconnect?
### Interconnect Buffers

- Allow “Software Pipelining”
  - Each data item
  - Spatial we would pipeline, running all three at once
  - Think of each schedule instance as one timestep in spatial pipeline.

### Add Precedence to SAT/ILP?

- Assign start time to each task
- **Precedence**: constrain start of each task to be greater than start+run of each predecessor
- **Time Exclusivity**: constrain non-overlap of start→start+run-1 on nodes on same PE
  - Maybe formulate as order on PE
  - And make PE order predecessor like a task predecessor?

### Memory Schedule Variants

- **Persistent**: holds memory whole time
  - E.g. task state, instructions
- **Task temporary**: only uses memory space while task running
- **Intra-Task**: use memory between point of production and consumption
  - E.g. Def-Use chains

### Branch-and-Bound
Brute-Force

- Try all schedules
- Branching/Backtracking Search
- Start w/ nothing scheduled (ready queue)
- At each move (branch) pick:
  - available resource time slot
  - ready task (predecessors completed)
  - schedule task on resource

Example

Branching Search

- Explores entire state space
  - finds optimum schedule
- Exponential work
  - \( O(N(\text{resources} \times \text{time-slots})) \)
- Many schedules completely uninteresting

Reducing Work

1. Canonicalize "equivalent" schedule configurations
2. Identify "dominating" schedule configurations
3. Prune partial configurations which will lead to worse (or unacceptable results)

“Equivalent” Schedules

- If multiple resources of same type
  - assignment of task to particular resource at a particular timeslot is not distinguishing

```
T1 T2 T3
T2 T3 T1
```

Keep track of resource usage by capacity at time-slot.
"Non-Equivalent" Schedule Prefixes

Pruning Prefixes?
- I'm not sure there is an efficient way (general)?
- Keep track of schedule set
  - walk through state-graph of scheduled prefixes
  - unfortunately, set is power-set so 2^N
  - ... but not all feasible, so shape of graph may simplify

Pruning
- If can establish a particular schedule path will be worse than one we've already seen
  - we can discard it w/out further exploration
- In particular:
  - LB = current schedule time + lower_bound_estimate
  - if LB greater than existing solution, prune

Dominant Schedules
- A strictly shorter schedule
  - scheduling the same or more tasks
  - will always be superior to the longer schedule

Pruning Techniques
Establish Lower Bound on schedule time
- Critical Path (ASAP schedule)
- Resource Bound
- Critical Chain

"Critical Chain" Lower Bound
- Bottleneck resource present coupled resource and latency bound

Single red resource
"Critical Chain" Lower Bound

- Bottleneck resource present coupled resource and latency bound

```
Critical path: 5
Resource Bound (1,1): 4
Critical Chain (1,1): 7
```

Single red resource

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Alpha-Beta Search

- Generalization
  - keep both upper and lower bound estimates on partial schedule
  - Lower bounds from CP, RB, CC
  - Upper bounds with List Scheduling
- expand most promising paths
  - (least upper bound, least lower bound)
- prune based on lower bounds exceeding known upper bound
  - (technique typically used in games/Chess)

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Alpha-Beta

- Each scheduling decision will tighten
  - lower/upper bound estimates
- Can choose to expand
  - least current time (breadth first)
  - least lower bound remaining (depth first)
  - least lower bound estimate
  - least upper bound estimate
- Can control greediness
  - weighting lower/upper bound
  - selecting "most promising"

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Note

- Aggressive pruning and ordering
  - can sometimes make polynomial time in practice
  - often cannot prove will be polynomial time
  - usually represents problem structure we still need to understand

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Multiple Resources

- Works for multiple resource case
- Computing lower-bounds per resource
  - resource constrained
- Sometimes deal with resource coupling
  - e.g. must have 1 A and 1 B simultaneously or in fixed time slot relation
  - e.g. bus and memory port

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Summary

- Resource estimates and Refinement
- SAT/ILP Schedule
- Software Pipelining
- Branch-and-bound search
  - "equivalent" states
  - dominators
  - estimates/pruning
Admin

• Class
  – Friday (no → holiday)
  – Next week:
    • Monday, Friday
    • No Wed.
• Next week’s reading all online

Big Ideas:

• Estimate Resource Usage
• Use dominators to reduce work
• Techniques:
  – Force-Directed
  – SAT/ILP
  – Coloring
  – Search
    • Branch-and-Bound
    • Alpha-Beta