Geometric Modeling: CS 175

Peter Schröder

Mo/Wed 10.30-12 Jorg 74

Geometric Modeling

Representation of geometric shapes in the computer

- mathematical foundations
- algorithms
**Geometric Modeling**

Goals
- model geometry of real world objects
  - car bodies, cells, turbulent flow, etc.
- model geometry of abstract objects
  - approximation theory
  - smoothness spaces

Model building
- interactive modeling
- data matching
  - build shapes from measurement
  - interpolation/approximation

Compute with it
- FEM simulation, evolving geometry
APPLICATIONS

Relevant for:
- engineering:
  - CAGD, simulation, visualization
- entertainment:
  - animation, games, virtual reality
- science:
  - medicine, biology

ISSUES

Time and space efficiency
- ideally linear complexity

Precision and consistency
- geometric and topologic errors

Flexibility
- large class of possible shapes
**INDUSTRY**

Manufacture from many parts
- precision
- consistency

**ENTERTAINMENT**

Shapes are not static
- need good controls (intuitive)

Animation
- flexibility, precision

Games
- speed, real-time
**Visualization**

Space efficiency
- huge data sets
- surface extraction
- volumetric data

---

**Representing Shapes**

Explicit: \[ y = f(x) \]
\[ y = -(x^2 - 1)^{1/2} \]
\[ 1 \leq x \leq 1 \]

Implicit: \[ F(x, y) = 0 \]
\[ x^2 + y^2 - 1 = 0 \]

Parametric: \( (x(t), y(t)) \)
\[ x = \cos(t) \quad y = \sin(t) \quad -\pi < t \leq \pi \]
SURFACES

Explicit
- terrains

Implicit
- changing topology
- volumetric

Parametric
- flexible
- easy to transform and discretize

COMPARISON

Explicit: not flexible enough

Implicit
- smooth shapes ("blobs")

Parametric
- very flexible
- easy to discretize
**Polygonal Models**

Polygon - universal primitive
- direct in hardware

Polygonal mesh
- collection of polygons

Quad mesh:
\[ \{(p_{n1}, p_{n2}, p_{n3}, p_{n4}) | n = 1, \ldots, N\} \]

**But** – not smooth

**Control Meshes**

Polygonal mesh – controls shape of a smooth model

User drags control vertices to change smooth shape
Geometric Primitives

Polygonal meshes
Parametric patches

Subdivision Meshes

Parametric Patches

Polynomials

\[ s^l(u, v) = A + Bu + Cv + Du^2 + Euv + Fv^2 \ldots \]

\[ s^r(u, v) = \ldots \]

How to match the two patches?
Subdivision

Smooth surfaces as the limit of a sequence of refinements

CS 175

Classic methods
- piecewise polynomials
- curves and surfaces
- splines, NURBS

State of the art methods
- subdivision curves and surfaces
Rules of the Game

Homeworks
- theory as well as programming

Project
- implementation of latest research and maybe beyond...