Introduction to

# Artificial Intelligence

## Lecture 4 – Adversarial search

CS/CNS/EE 154

Andreas Krause

TexPoint fonts used in EMF

# Projects

- Recitations: Thursday 4:30pm – 5:30pm, Annenberg 107
  - Details about projects
  - Will also be posted on webpage
- By Monday 10/11
  - Form team of 3 students
  - Need to select project (Doodle link will be sent today)
  - For independent projects: need to submit proposal
- If you don't have a team, send email to TAs
- Homework 1 out on Friday

# Types of games

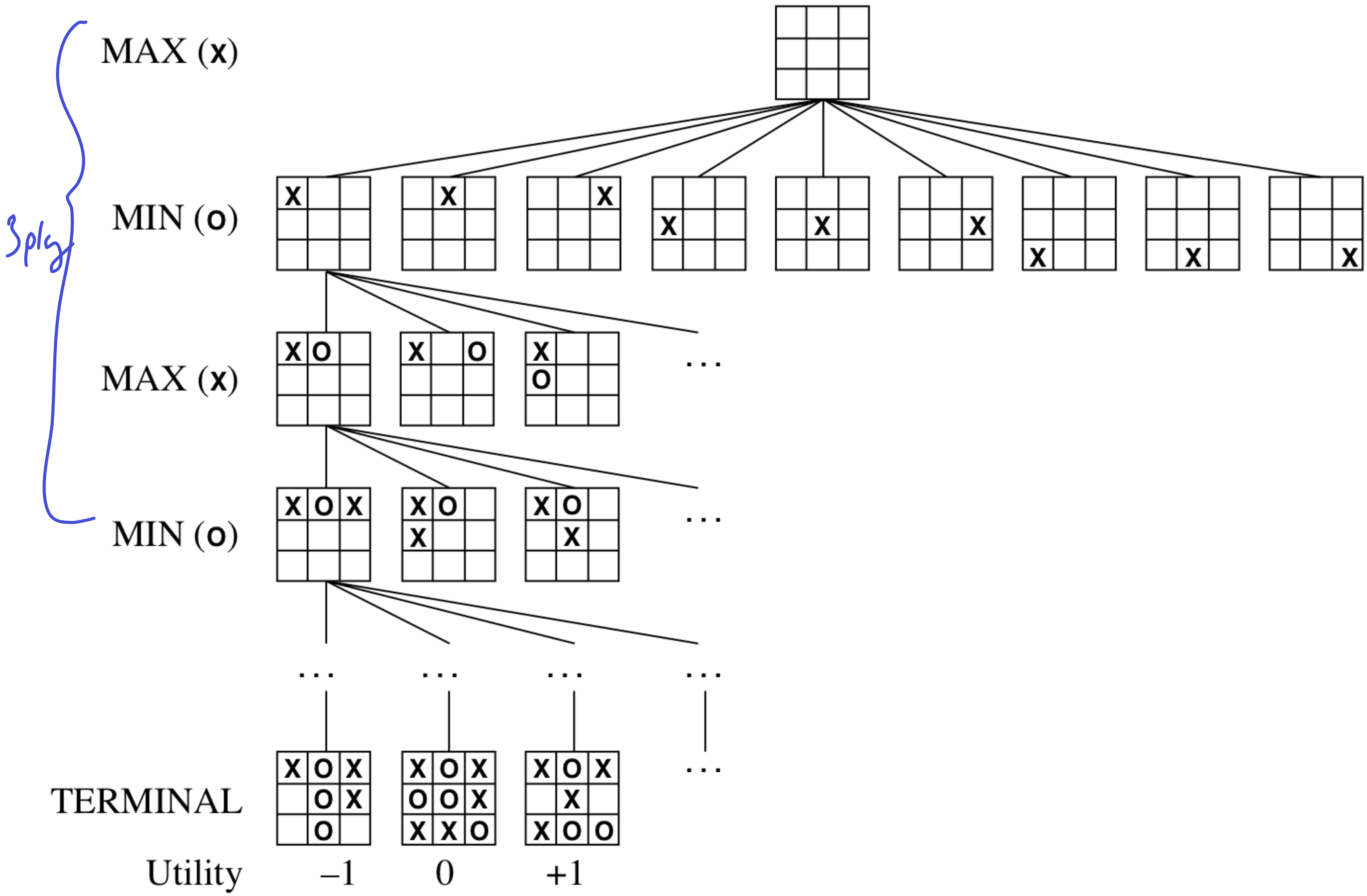
| | Chess | Backgammon | Poker | Rock Paper Scissors | WoW |
|---|---|---|---|---|---|
| **Observable?** | Y | Y | N | Y * | N |
| **Determ.?** | Y | N | N | | N |
| **Simultan.?** | N | N | N | Y | Y |
| **Zero-sum?** | Y | Y | Y | Y | N |
| **Discrete?** | Y | Y | Y | Y | N |
| **# Players?** | 2 | 2 | $\geq 2$ | 2 | $\approx 8 \cdot 10^6$ |

In this class, focus on two-player, sequential, zero-sum, discrete (mostly deterministic)

# Games vs. search

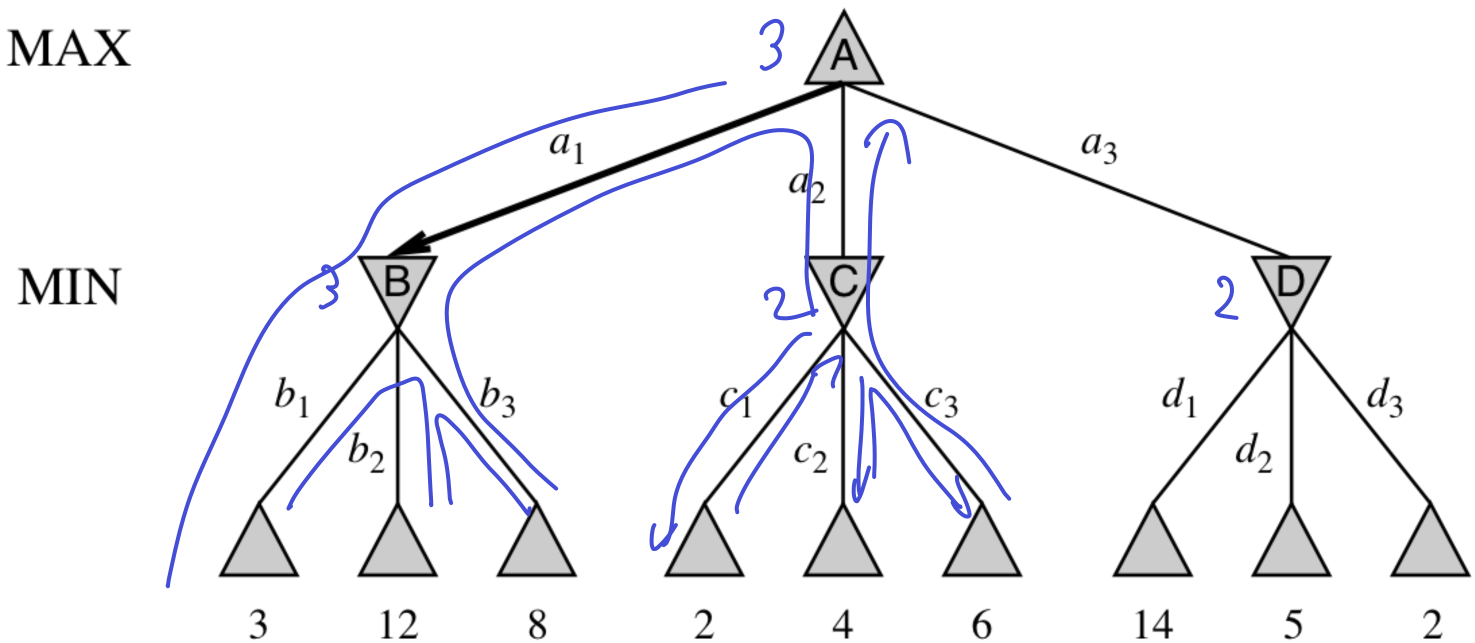
- In games, actions are nondeterministic
  - Opponent can affect state of the environment
- Optimal solution no longer sequence of actions, instead a strategy (policy, conditional plan)
  - If you X I'll do Y, else if you do Y I'll do Z, ….

# Game tree



# Minimax game tree

- Search for optimal move no matter what opponent does
- minimax value = best achievable payoff against best play




Can evaluate using DFS

# Solving deterministic games

- MiniMax used to calculate optimal move:
- Inductive definition:

If $n$ is terminal node:

- Value is utility($n.state$)

If $n$ is MAX node:

- Value is highest value of all successor node values

If $n$ is MIN node

- Value is lowest value of all successor node values

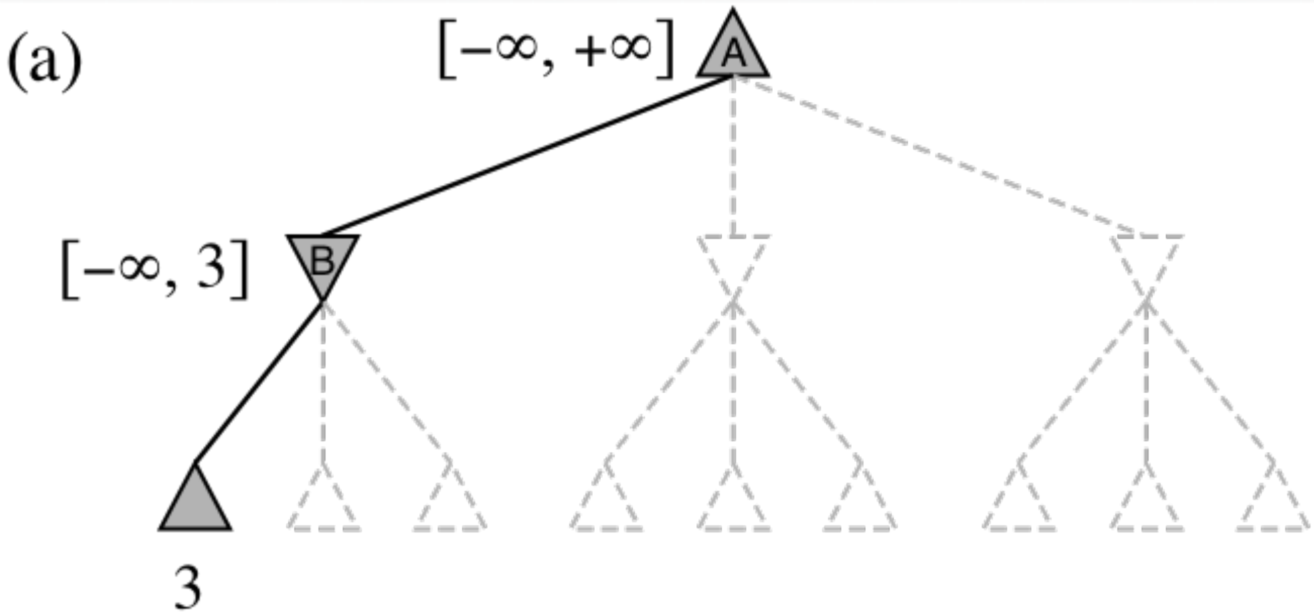
# Properties of minimax search

- Complete? Yes if tree is finite
- Time complexity? $O(b^m)$
- Space complexity? $O(b \cdot m)$
- Optimal? Yes if finite and if opponent plays optimally

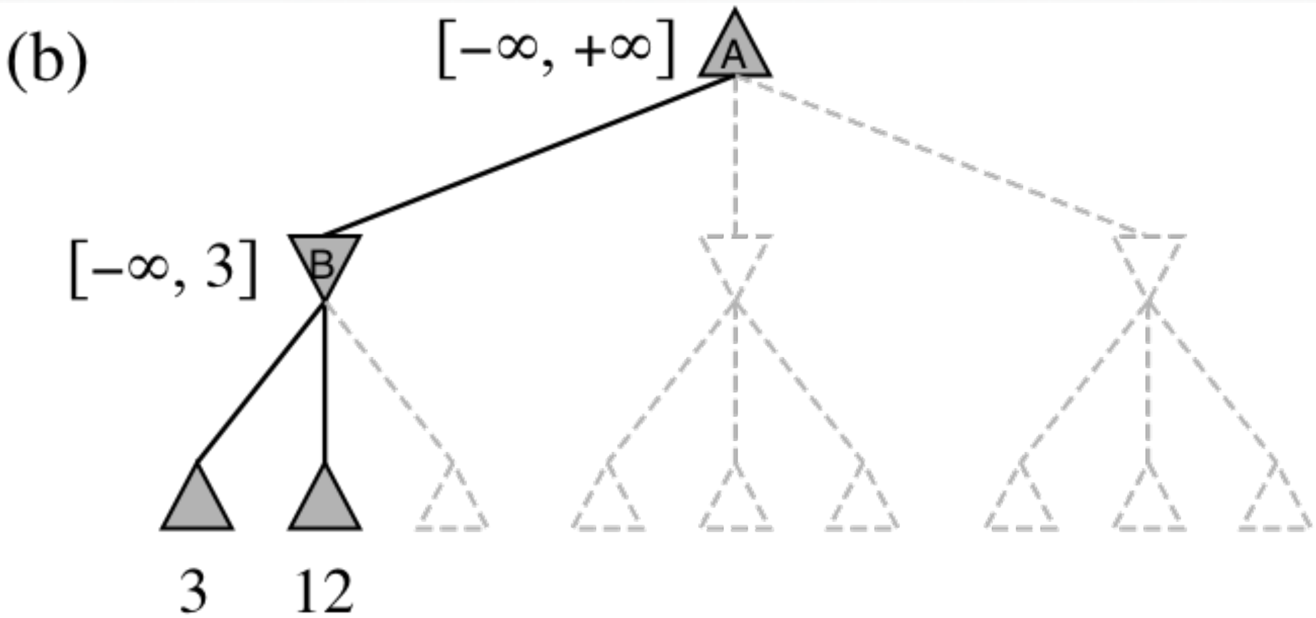
For TicTacToe: $b \leq 9$, $m \leq 9$ $9^9$; elim symmetrics $\Rightarrow \leq 30\,000$ nodes

For Chess: $b \approx 35$, $m \approx 50$, $35^{50}$ hopelessly intractable :(

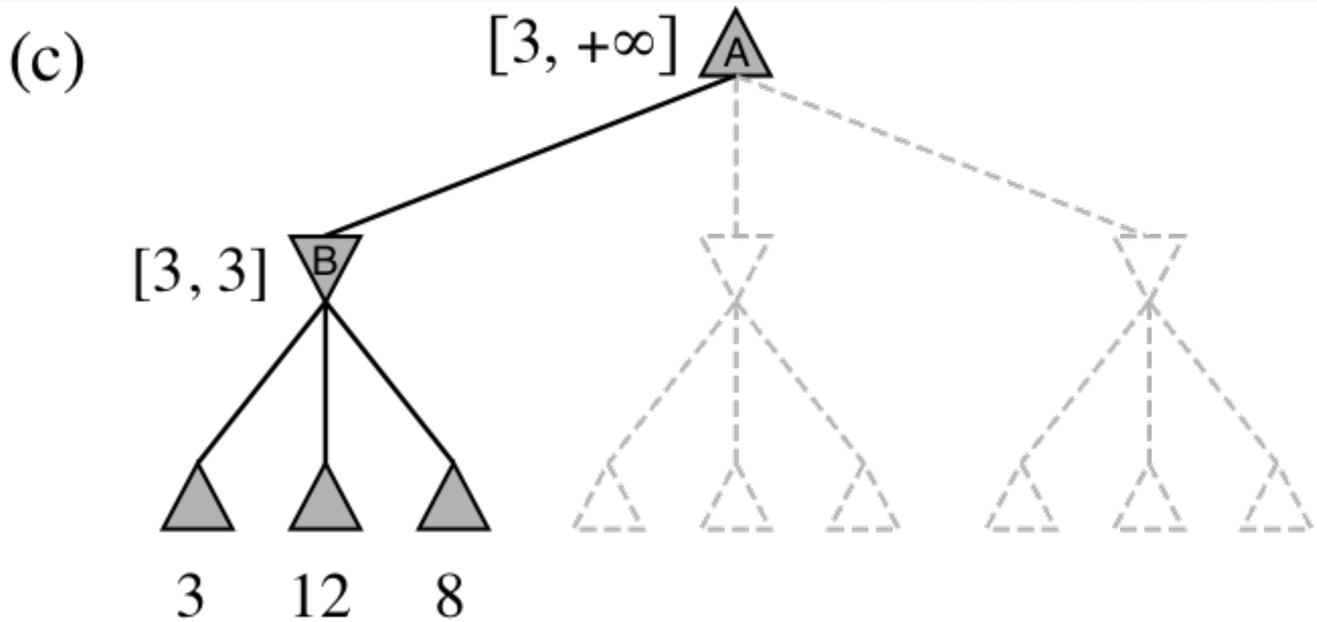
# α-β-pruning



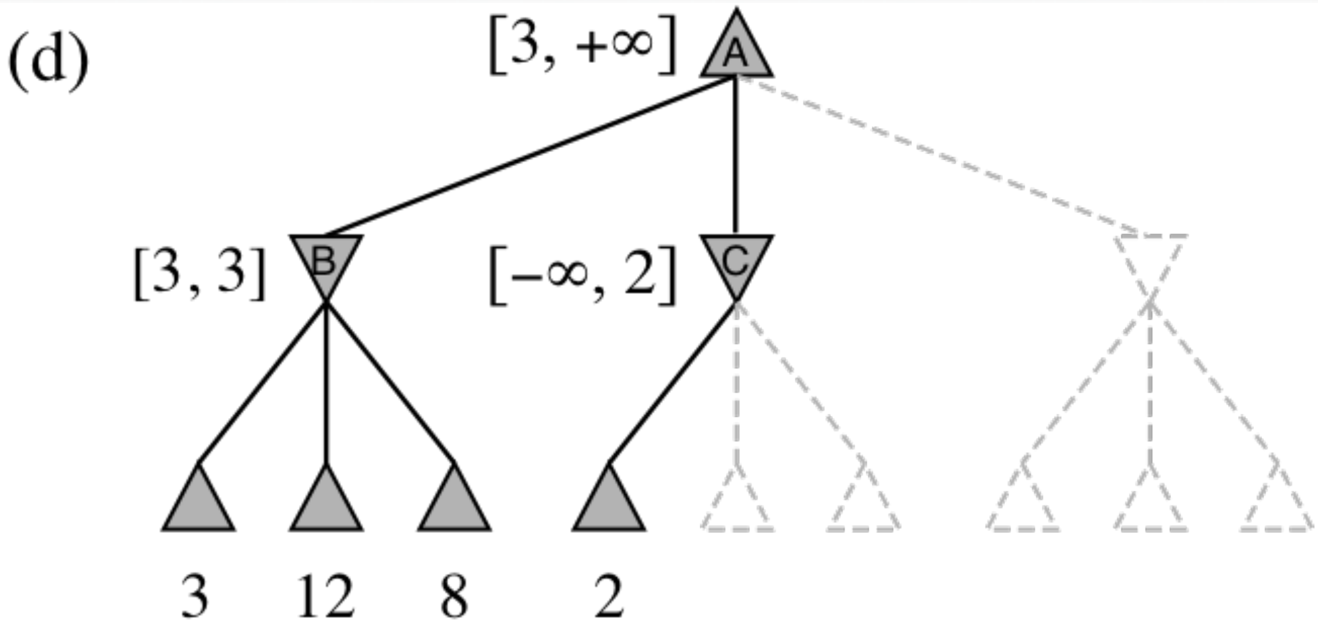
# α-β-pruning



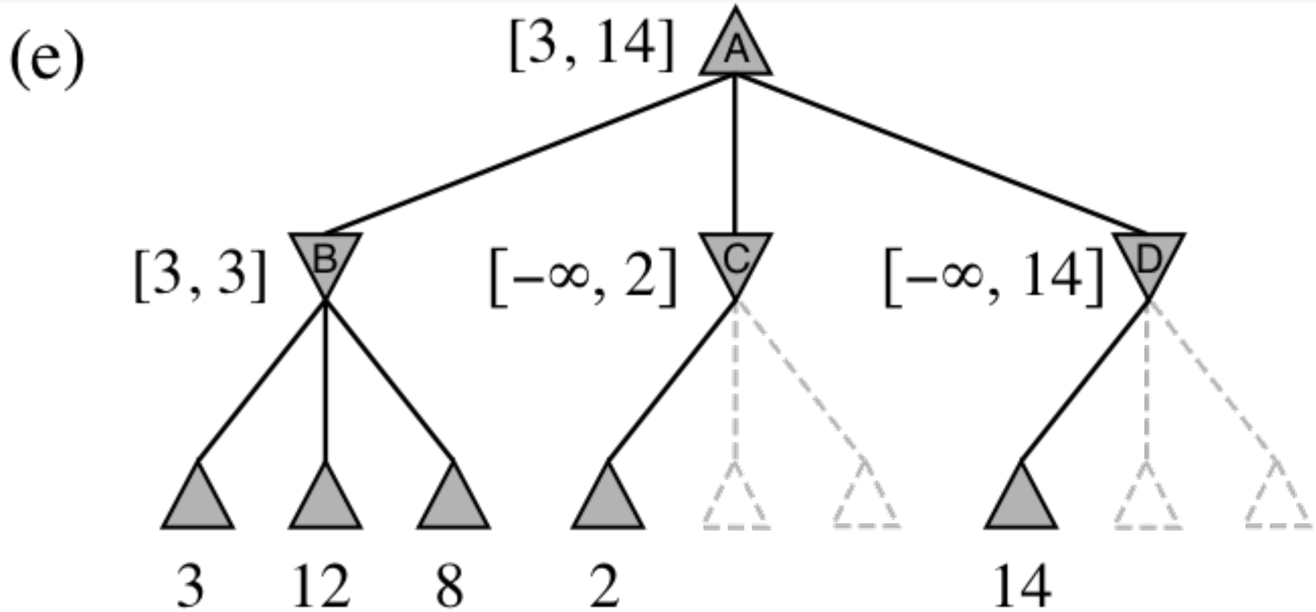
# α-β-pruning



# α-β-pruning

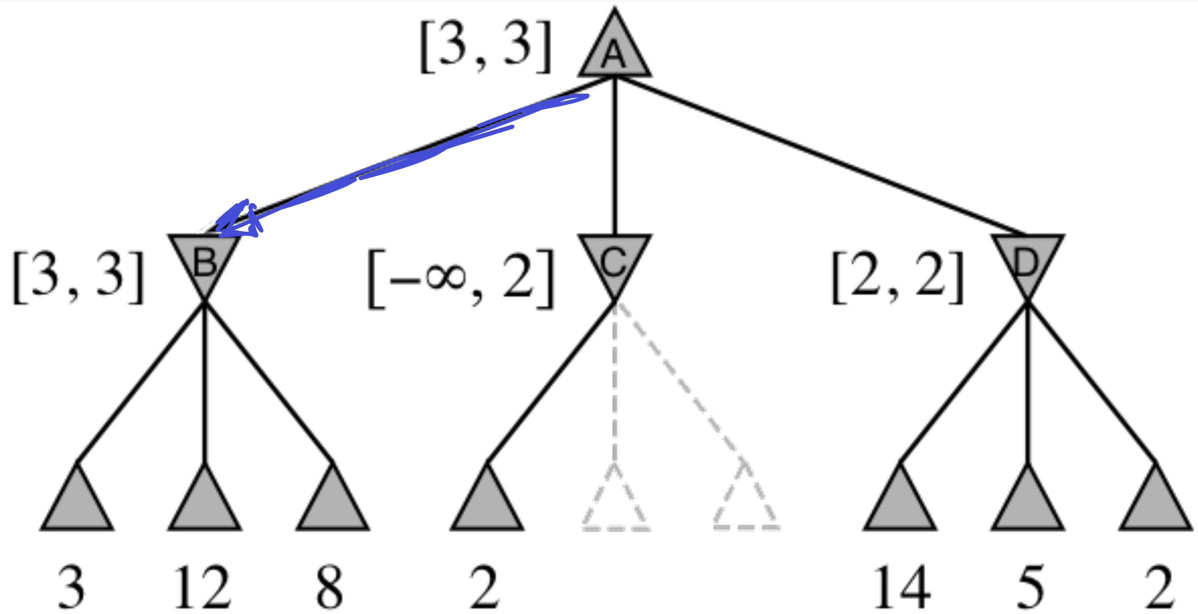


# α-β-pruning



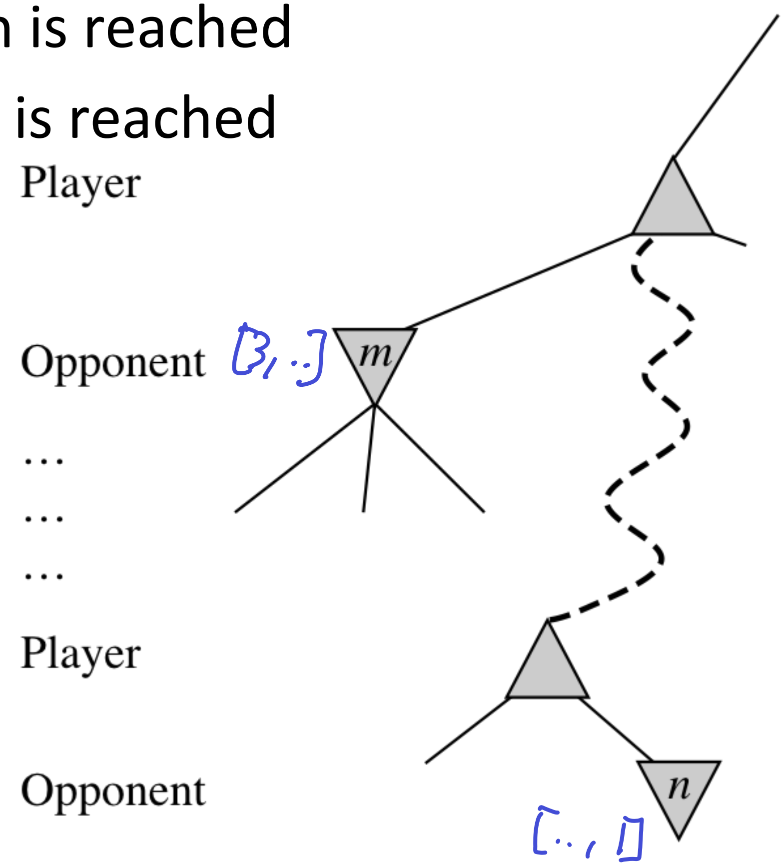
# α-β-pruning

(f)



# α-β-pruning

- Key idea: For each node n in minimax tree keep track of
  - α: Best value for MAX player if n is reached
  - β: Best value for MIN player if n is reached
- Never need to explore consequences of actions for which β<α
- Avoid exploring "provably suboptimal" parts of minimax tree



# α-β-pruning algorithm

**function** ALPHA-BETA-DECISION(*state*) **returns** an action
  **return** the *a* in ACTIONS(*state*) maximizing MIN-VALUE(RESULT(*a*, *state*))

---

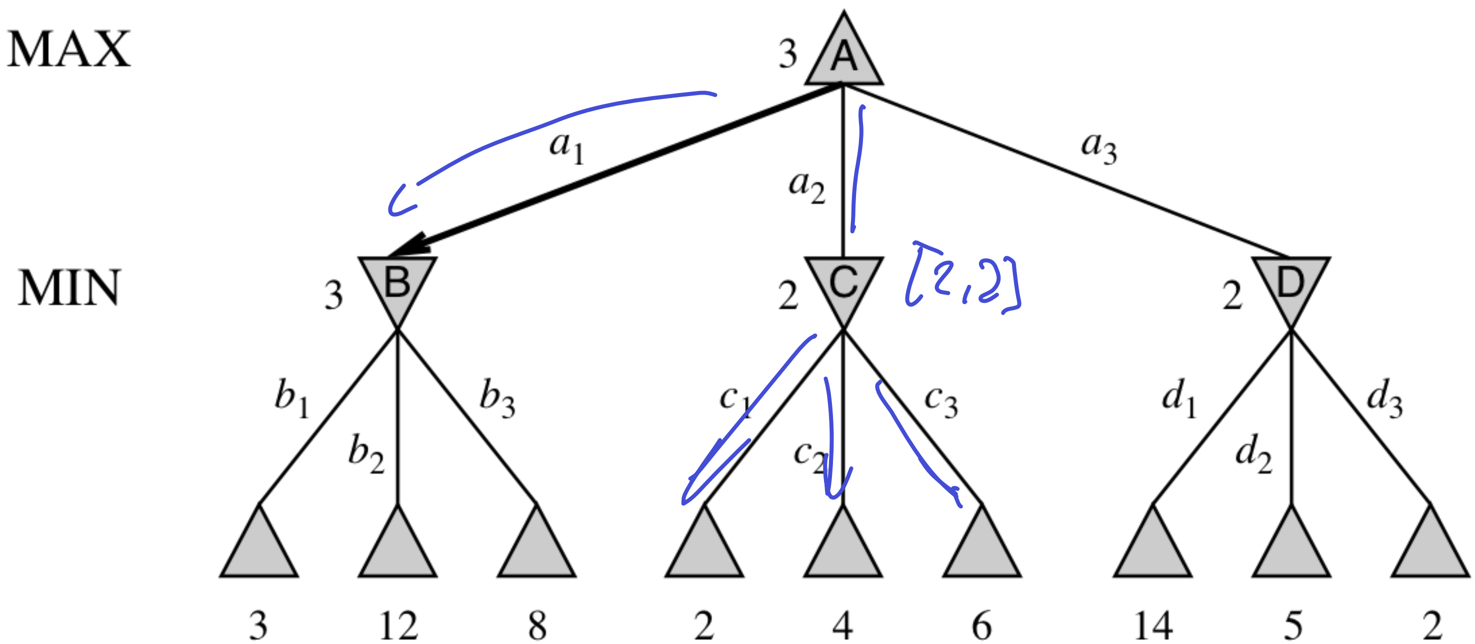
**function** MAX-VALUE(*state*, $\alpha$, $\beta$) **returns** *a utility value*
  **inputs**: *state*, current state in game
    $\alpha$, the value of the best alternative for MAX along the path to *state*
    $\beta$, the value of the best alternative for MIN along the path to *state*

  **if** TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  **for** *a*, *s* in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MAX($v$, MIN-VALUE($s, \alpha, \beta$))
    **if** $v \geq \beta$ **then return** $v$
    $\alpha \leftarrow$ MAX($\alpha$, $v$)
  **return** $v$

---

**function** MIN-VALUE(*state*, $\alpha$, $\beta$) **returns** *a utility value*
  same as MAX-VALUE but with roles of $\alpha$, $\beta$ reversed

# Does move ordering matter?



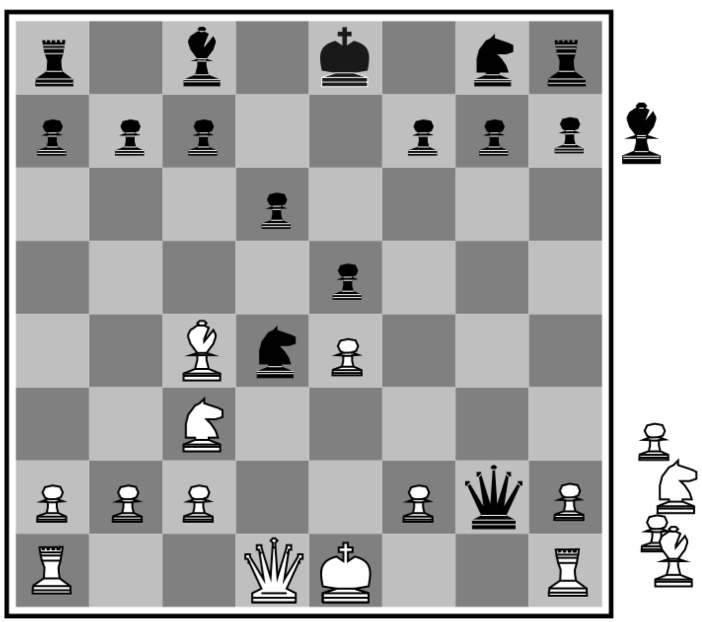
# Move ordering matters a lot

- Worst case: No improvement $O(b^m)$
- Best case (ideal ordering): $O(b^{m/2})$ ← can get fairly close in practice
- Random ordering: $O\left((b/\log b)^m\right)$
- How to find a good ordering?

Use IDS, "remember" best moves from shallower trees

# Large state spaces

- Typical branching factor in chess: 35
- Computing the complete minimax tree is intractable
- Instead: Cut off search, and replace utility(s) with eval(s)
  - eval(s) is heuristic value of state s



Materia ( # black / white pawns, rooks, ... )

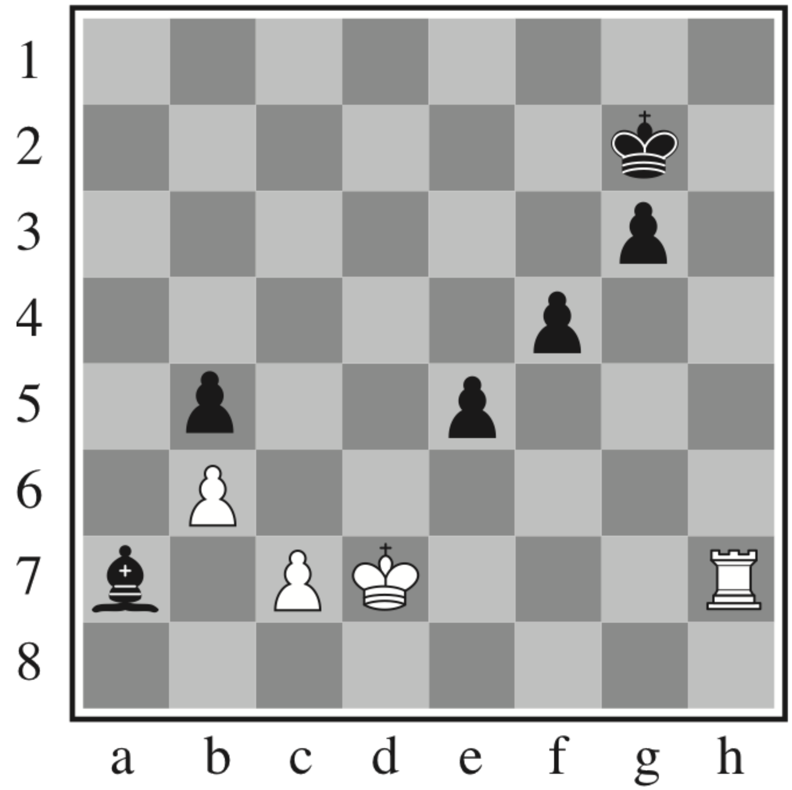
# Developing evaluation functions

- This is where expert knowledge comes in
- Typical approach:
  - Select features $f_1,\ldots,f_n$ that may be useful, e.g., value of pieces on board, positions of pieces, …
  - Learn weights from examples

$$eval(s) = \sum_{i=1}^{n} w_i f_i(s)$$

  - Deep Blue used ~6,000 different features!
  - Often, reinforcement learning is very useful here (e.g., TD-gammon beats world champion in backgammon)

# Problems with cutoff search




Black to move

# Taming the horizon effect

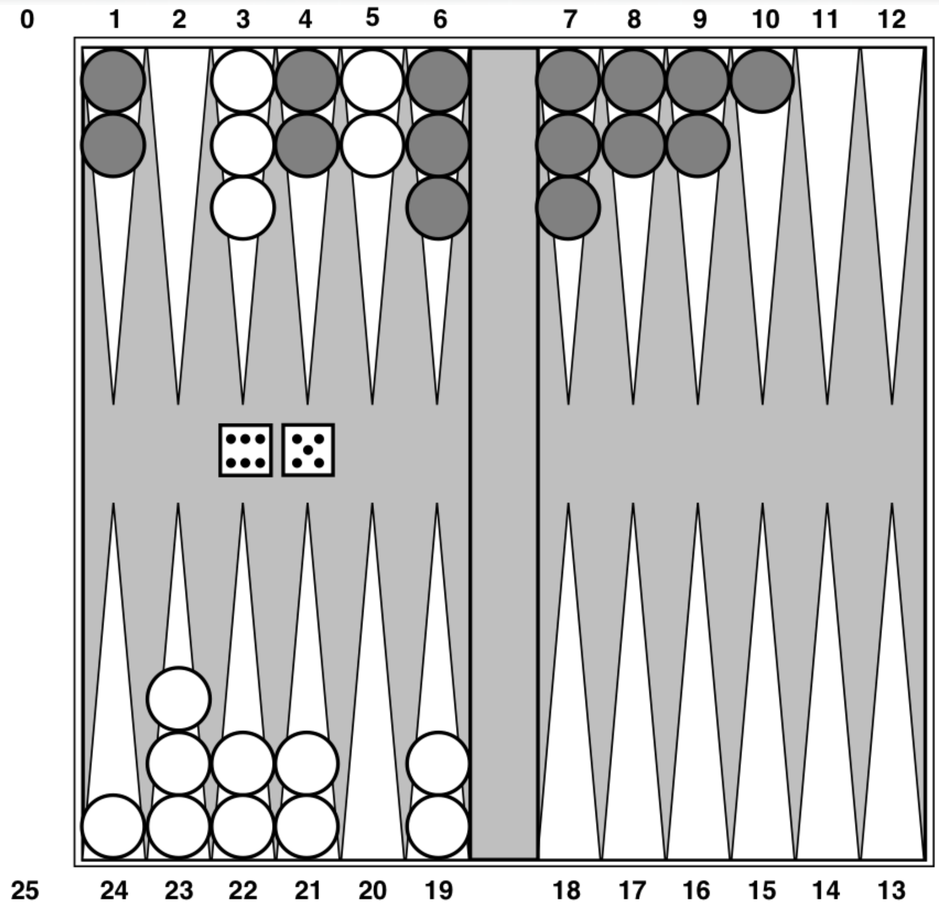
- Quiescence search
  - Evaluation function also evaluates "stability" (e.g., strong captures, etc.)
  - Cutoff postponed if position is unstable
  - Search time no longer constant
- Singular extension
  - Search deeper if a node's value is much better than its siblings'
  - Reduces effective branching factor
  - Can search much longer sequences (even 30-40ply)

# Playing world class chess

- Current PCs can evaluate ~200 million nodes / 3 min
- Minimax search: ~5 ply lookahead
- With α-β pruning: ~10 ply

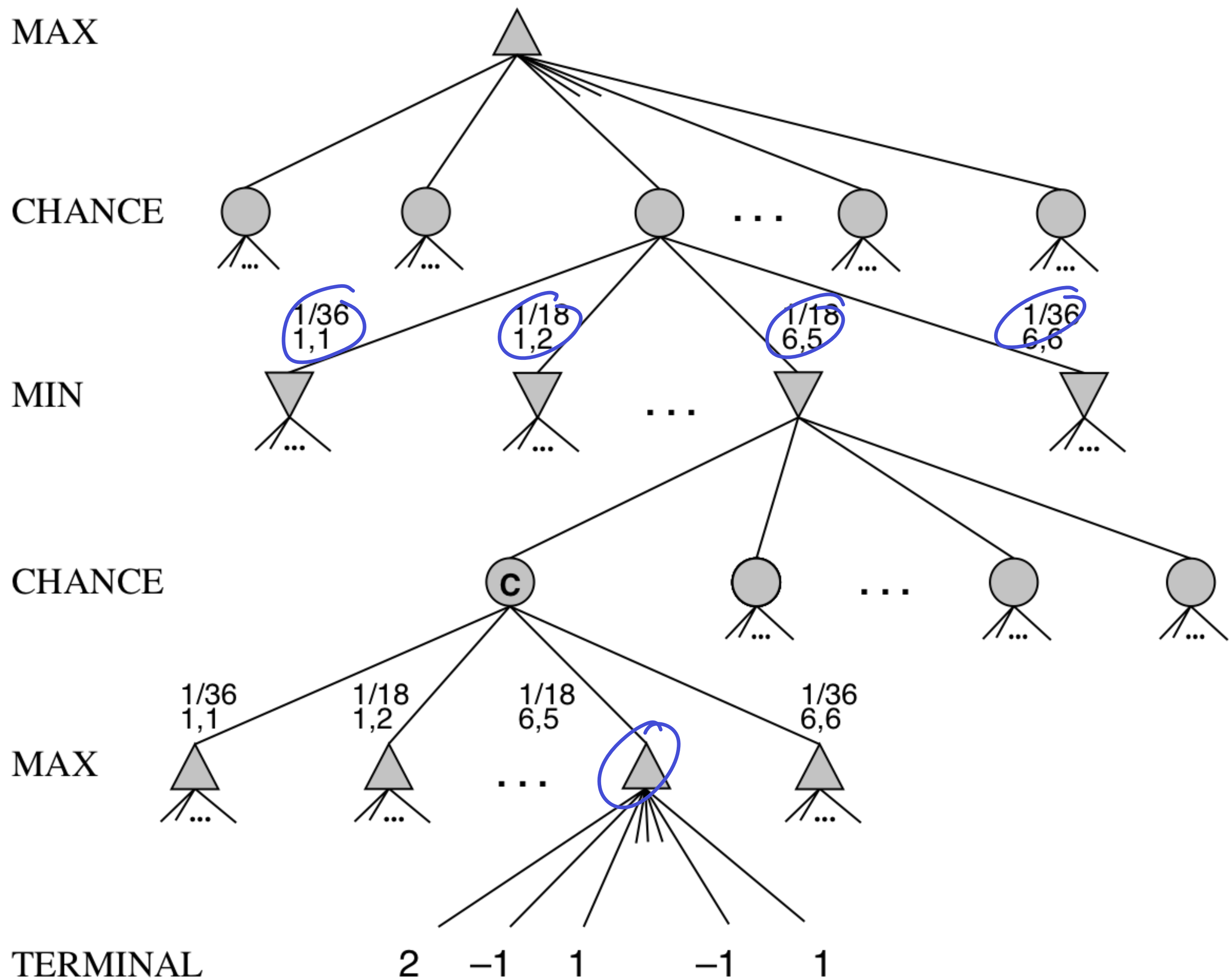
- Further improvements:
  - *Quiescence search*: Only evaluate "stable" positions
  - *Transposition tables*: Remember states evaluated before
  - *Singular extensions*: Expand tree if there is singular best move
  - *Null move heuristic*: Get lower bound by letting opp. move 2x
  - *Precompute endgames* (all 5, some 6 piece positions)
  - *Opening library* (up to ~30ply in first couple moves)
- Hydra: 18 ply lookahead (on 64 processor cluster)

# Stochastic games




- Two "types" of uncertainty
- Adversarial and stochastic

# ExpectiMiniMax tree



# Solving stochastic games

- ExpectiMiniMax used to calculate optimal move
- Defined inductively:

If *n* is terminal node (or cutoff):

- Value is utility(*n.state*) (or eval(*n.state*))

If *n* is MAX node:

- Value is highest value of all successor node values

If *n* is MIN node

- Value is lowest value of all successor node values

If *n* is CHANCE node

- Value is (weighted) average of all successor node values

# Dealing with large state spaces

- Backgammon:
  - 21 possible roles with 2 die; ~20 legal moves
  - #nodes for depth 4 tree:

$21\,(20 \cdot 21)^3$

- As depth increases, reaching any particular node becomes exponentially unlikely
  - Lookahead becomes less valuable
  - α-β-pruning much less useful: world just won't play along!

- TD-gammon competitive with best human players:
  - Uses only 2 ply lookahead!
  - But very carefully trained evaluation function