

CS/CNS/EE 154: Artificial Intelligence
Problem Set 2

Handed out: 29 Oct 2010
Due: 10 Nov 2010

1 First Order Logic [25 points]

- (i) Write down a satisfiable logical sentence in First Order Logic, such that every world in which it is true, contains exactly one object.
- (ii) Now, consider a vocabulary that contains the following symbols:

Occupation(p, o) : Predicate. Person p has occupation o .

Customer($p1, p2$) : Predicate. Person $p1$ is a customer of person $p2$

Boss($p1, p2$) : Predicate. Person $p1$ is a boss of person $p2$.

Doctor, Surgeon, Lawyer, Actor: Constants denoting occupations.

Emily, Joe: Constants denoting people.

Use these symbols to write the following assertions in First Order Logic:

- (a) Emily is either a surgeon or a lawyer
- (b) Joe is an actor, but he also holds another job.
- (c) All surgeons are doctors.
- (d) Joe does not have a lawyer (i.e. Jose is not a customer of any lawyer)
- (e) Emily has a boss who is a lawyer.
- (f) There exists a lawyer whose customers are all doctors.
- (g) Every surgeon has a lawyer

2 Conditional Probabilities [25 points]

For each statement below, either prove it is true, or give a counterexample showing it is false.

- (a) If $P(a|b, c) = P(b|a, c)$, then $P(a|c) = P(b|c)$
- (b) If $P(a|b, c) = P(a)$, then $P(b|c) = P(b)$
- (c) If $P(a|b) = P(a)$, then $P(a|b, c) = P(a|c)$

3 Finding the fake coin [25 points]

Suppose you are given a bag containing n unbiased coins. You are also told that $n - 1$ of these coins are normal, that is, they have a head on one side and a tail in the other. The remaining one is fake and has heads on both of its sides.

- (a) Suppose you pick one coin from the bag at random and then you flip it and get a head. Given this result, what is the probability that the coin you choose is the fake coin? (Note we are asking for a conditional probability).
- (b) Suppose you continue flipping the same coin for a total of k times and you see k heads. Now what is the conditional probability that you picked the fake coin?
- (c) Now suppose you devise the following method to determine if the coin is fake or not. You flip it k times, and your method returns it is a fake coin if all k flips result in heads. Otherwise the method returns that it is a normal coin. What is the probability that this method returns an error? (note this time we ask for an unconditional probability)

4 Bayesian Networks [25 points]

Suppose you have a bag of three biased coins a , b , and c , with probabilities of coming up heads of 0.2, 0.6, and 0.8 respectively. One coin is drawn randomly from the bag (with equal likelihood of drawing each of the coins). Now, you flip the coin three times to generate the sequence of outcomes X_1 , X_2 , and X_3 .

- (a) Draw the Bayesian Network corresponding to this setup and define the necessary Conditional Probability Table (CPT).
- (b) Calculate which coin was most likely to have been drawn from the bag, if two of the observed outcomes in the sequence were heads and the other was a tail.