CMS/EE 144 **Gurus: Carly/Irene**

HW 5: Game Theory Warmup

Assigned: 02/15/17 Due: 02/22/17 by 10:30am

We encourage you to discuss "collaboration problems" with others, but you need to write up the actual solutions alone. At the top of your homework sheet, please list all the people with whom you discussed. Crediting help from other classmates will not take away any credit from you. For "no-collaboration" problems, you are expected to work alone.

*You must turn in your homework electronically via Moodle. Please submit a **single file** in zip or tar.gz format, which includes your answers and any code, titled **LastName-FirstName.zip** or **LastName-Firstname.tar.gz**.*

Start early and come to office hours with your questions! We also encourage you to post your questions on Piazza, as well as answer the questions asked by others on Piazza. For no-collaboration problems, only clarification questions should be posted to Piazza.

1 Collaboration Game (50 points, collaboration allowed)

Consider the two-player game described by the payoff matrix in Figure 1. The rows and columns correspond to player A's and player B's actions respectively. Each cell contains player A's payoff, followed by player B's payoff.

For all of these questions, justify your answer's correctness with a short bit of reasoning and/or show the work done to get the answer.

	L	R
T	5, 2	0, 0
B	0, 0	3, 4

Figure 1: Payoff matrix

1. **[10 points]** Find the dominant strategies, if any.
2. **[10 points]** Find all the pure Nash equilibria. What are their payoffs for each player?
3. **[10 points]** Find the mixed Nash equilibrium. What is its expected payoff for each player?
4. **[10 points]** *Coordination.* Suppose an external randomizing device determines what the players should play in order to maximize their payoff. If a randomizing device picks each pure Nash equilibrium with equal probability, and the players follow the orders of the randomizing device, what is the expected payoff of each player?
5. **[10 points]** Does either player have an incentive to disobey the randomizing device? That is, assuming Player B obeys the randomizing device, does Player A ever want to play something else? And vice versa?

2 Mafia internal affairs [25 points, collaboration allowed]

Consider a Mafia clan with 4 members. There is a strict hierarchy between the members, as shown in Figure 2. The clan has just made $\$10,000$ and the time has come to divide up the loot between the members.

The rules for loot redistribution are the following. First, the highest member in the hierarchy proposes an allocation and this allocation is put to vote. If the allocation is approved (by majority or tie), it gets implemented. Else, the highest ranking member (who has now lost the approval of the clan) is killed and the next highest member of the hierarchy gets to propose an allocation. This process continues until an allocation gets approved.

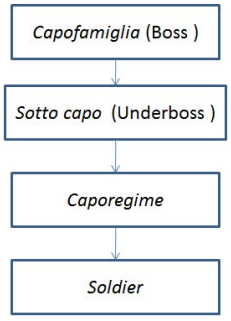

Figure 2: Mafia hierarchy

Each clan member is assumed to vote with the following considerations in decreasing order of priority: (i) stay alive, (ii) make as much money as possible, (iii) money remaining constant, kill off as many higher ranking members as possible. You may assume that the smallest unit of money is $\$1$.

What allocation should the boss propose? Interpret this allocation. Does it agree with your intuition?

3 Alternating offers [25 points, collaboration allowed]

Adam has made a deal with Carly and Irene regarding the grading of HW5. If they manage to finish grading all HW5 before the deadline, they will receive a bonus of $\$5$ in total. Of course, the need to cooperate to complete the grading, so they need to figure out how to divide the bonus if they're successful.

To do this, the agree on the following scheme. Carly first makes an offer to Irene. Her offer could be any number of $\{\$0, \$1, \$2, \$3, \$4, \$5\}$. Irene can then either accept the offer or decline it. If Irene accepts the offer, she will get that amount of money and Carly will take the rest. If Irene declines the offer, then Irene can counter-offer to Carly following the same rules. In order to save enough time for grading, they agree to stop after N offers have been made in total. If these N offers are all rejected, they will not grade HW5 and thus receive no bonus.

To given you a feel for this game, consider the case of $N = 1$. Here, only Carly can make an offer. In this case, the following is an equilibrium. Irene only accepts $\$4, \$5$ and rejects $\$0, \$1, \$2, \$3$. Carly offers $4. This is an equilibrium profile because neither Carly nor Irene can possibly earn more if one of them is allowed to change the strategy while the other's strategy is fixed.

Your task:

(a) **[10 points]** If $N = 88$, find the best strategy for Irene.

(b) **[10 points]** If $N = 99$, find the best strategy for Irene.