2 Ethernet

2.1 W&P, P3.2

3 Points. Consider the Slotted ALOHA MAC protocol. There are N nodes sharing a medium, and time is divided into slots. Each packet takes up a single slot. If a node has a packet to send, it attempts transmission with a certain probability. The transmission succeeds if no other node attempts transmission in that slot.

Now, suppose that we want to give differentiated services to these nodes, i.e., we want different nodes to get a different share of the medium. The scheme we choose works as follows: If node i has a packet to send, it will try to send the packet with probability p_i. Assume that every node has a packet to send all times. In such a situation, will the nodes indeed get a share of the medium in the ratio of their probability of access?

2.2 W&P, P3.4

4 Points. Consider a commercial 10 Mbps Ethernet configuration with one hub (i.e., all end stations are in a single collision domain).

(a) **2 Point.** Find the Ethernet efficiency for transporting 512 byte packets (including Ethernet overhead) assuming that the propagation delay between the communicating end stations is always 25.6 μs, and that there are many pairs of end stations trying to communicate.

(b) **2 Points.** Recall that the maximum efficiency of Slotted Aloha is $1/e$. Find the threshold for the frame size (including Ethernet overhead) such that Ethernet is more efficient than Slotted Aloha if the fixed frame size is larger than this threshold. Explain why Ethernet becomes less efficient as the frame size becomes smaller.

2.3 W&P, P3.5

4 Point. Ethernet standards require a minimum frame size of 512 bits in order to ensure that a node can detect any possible collision while it is still transmitting. This corresponds to the number of bits that can be transmitted at 10 Mbps in one roundtrip time. It only takes one propagation delay, however, for the first bit of an Ethernet frame to traverse the entire length of the network, and during this time, 256 bits are transmitted. Why, then, is it necessary that the minimum frame size be 512 bits instead of 256?

2.4 Switch vs Hub, W&P, P3.7

6 Points. In the network shown in Figure 1, all of the devices want to transmit at an average rate of R Mbps, with equal amounts of traffic going to every other node. Assume that all of the links are half-duplex and operate at 100Mbps and that the medium access control protocol is perfectly efficient. Thus, each link can only be used in one direction at a time, at 100Mbps. There is no delay to switch from one direction to the other.

1. **3 Points.** What is the maximum value of R?
2. **3 Points.** The hub is now replaced with another switch. What is the maximum value of R now?

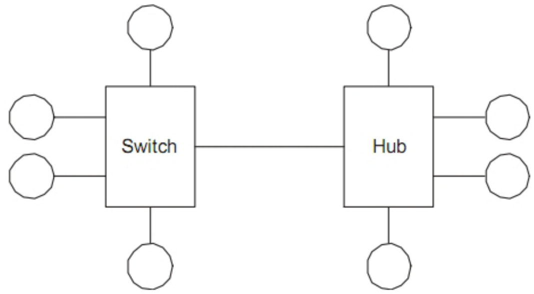

Figure 1: An ethernet. Each circle represents a host, that wants to send an aggregate of R Mbps traffic, evenly to other hosts.

2.5 Spanning Tree Protocol, W&P, P3.6

6 Points. Consider the network topology shown in Figure 2, where $1, 2, \ldots, 8$ denote 8 switches interconnecting 9 Ethernets.

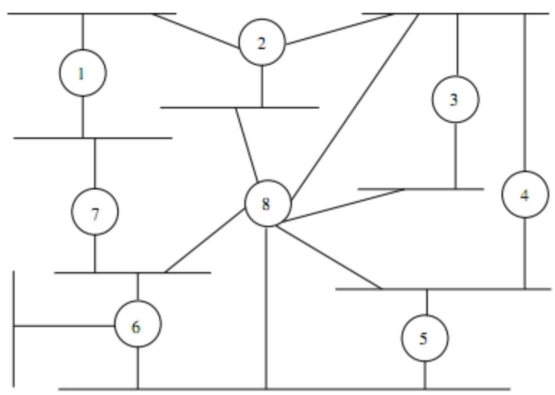

Figure 2: Each circle represents a switch, interconnecting 9 Ethernets.

1. **3 Points.** Determine which links get deactivated after the Spanning Tree protocol runs, and indicate them on the diagram by putting a small X through the deactivated links.
2. **3 Points.** A disgruntled employee wishes to disrupt the network, so she plans on unplugging central bridge switch 8. How does this affect the spanning tree and the paths that Ethernet packets follow?

2.6 Aloha

(a) (Equal sharing). Assume that n hosts share a medium using the slotted ALOHA protocol: in every time slot, each host attempts to send a packet with probability p. A host succeeds in sending a packet in a time slot if and only if it is the only host that sends a packet in that time slot.

1. **2 Points.** What is the probability that a given host sends a packet successfully in a give time slot?
2. **2 Points.** What is the probability $\mathbb{P}\{\text{a packet sent successfully}\}$ that a packet (from any host) is sent successfully in a given time slot?
3. **4 Points.** What choice of p maximizes the probability $\mathbb{P}\{\text{a packet sent successfully}\}$? How does this maximum probability behave as $n \to \infty$?

(b) (Unequal sharing). Assume that n hosts share a medium using the slotted ALOHA protocol, but at every time slot, each host attempts to send a packet with a probability that may be different for different hosts. More specifically, let $\mathcal{N} := \{1, \ldots, n\}$ denote the collection of hosts and assume that host i attempts to send a packet with probability $p_i \in (0, 1)$ at every time slot for $i = 1, \ldots, n$.

1. **2 Points.** What is the probability $P_i := \mathbb{P}\{i \text{ sends a packet successfully}\}$ that host $i \in \mathcal{N}$ sends a packet successfully at a given time slot?
2. **2 Points.** What is the ratio $P_1 : P_2 : P_3 : \cdots : P_n$? This ratio characterizes the share of medium among the hosts. Is the share of medium proportional to the probabilities p_i that hosts attempt to send packets, i.e., is the ratio $P_1 : P_2 : P_3 : \cdots : P_n$ equal to $p_1 : p_2 : p_3 : \cdots : p_n$?
3. **(*) 5 Points.** Assume $\sum_{i=1}^n p_i = 1$ and let $P := \sum_{i=1}^n P_i$ denote the probability that a packet gets successfully transmitted (by any host) at a given time slot. Prove that $(p_1, p_2, p_3, \ldots, p_n) = (1/n, 1/n, \ldots, 1/n)$ minimizes P, i.e., $(1/n, 1/n, \ldots, 1/n)$ is the solution to

$$\begin{aligned} \min \quad & \sum_{i=1}^n p_i \prod_{j \neq i} (1 - p_j) \\ \text{s.t.} \quad & 0 \le p_i \le 1, \quad i = 1, \ldots, n; \\ & \sum_{i=1}^n p_i = 1. \end{aligned}$$

   It means that equal sharing of the medium minimizes the throughput.

3 Routing

3.1 Longest prefix matching (exercise)

1 Point. Consider the following routing table.

IP	output port
166.111.8.0/24	1
166.111.0.0/16	2

Which outport should a packet be forwarded to, if its destination IP address is 166.111.8.28?

3.2 Static routing, W&P, P5.1

Consider the network topology depicted in Figure 3. Each link is marked with its weight/cost.

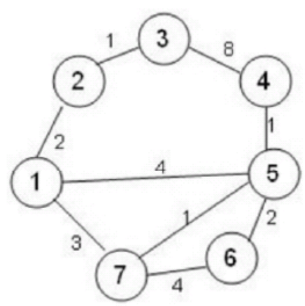

Figure 3: Network topology with link weights.

(a) **3 points.** Run Dijkstras algorithm on the above network to determine the **routing table** for node 3. Please show **steps** of the algorithm.

(b) **3 points.** Repeat (a) using Bellman-Ford algorithm.

3.3 Dynamic routing

Consider 5 stations connected in a bi-directional ring, as shown in Figure 4. Suppose station 0 is the only sender, and it sends packets to all other stations 1, 2, 3, 4 at rates 4, 3, 2, 1 packets/sec, respectively. Note that these are end-to-end traffic rates between source 0 and all destinations, not the link flow rates which depend on the routing. These end-to-end source-to-destination rates and the routing decision jointly induce a traffic pattern on the network and hence flow rates on the links.

(a) **3 points.** Table 1 shows the routing tables at each station. For each station, the first column is D (destination) and the second column is NN (next node). Indicate in a diagram the flow rates on the links as implemented by the routing table.

(b) **3 points.** Use the link flow rates obtained in (a) as the links costs (note that there are 10 links in total). Fix those link costs, and use the Dijkstra algorithm (and show the steps) to compute the new shortest paths (with minimum cost) from station 0 to all other stations, and calculate the new link rates using the new shortest paths.

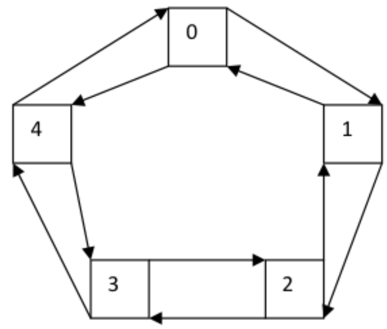

Figure 4: A bi-directional ring topology.

Table 1: Routing tables of stations

Station 0		Station 1		Station 2		Station 3		Station 4	
D	NN	D	NN	D	NN	D	NN	D	NN
1	1	0	2	0	3	0	4	0	0
2	1	2	2	1	3	1	4	1	0
3	1	3	2	3	3	2	4	2	0
4	1	4	2	4	3	4	4	3	0

(c) **2 points.** Use the links flow rates you obtained in (b) as the links costs. Again, fix those link costs, and compute the new shortest paths from station 0 to all other stations, using the Bellman-Ford algorithm (and show the steps), and calculate the new link rates using the new shortest paths.

(d) **1 point.** If this procedure is repeated, will the routing ever converge?

3.4 Dynamic routing (exercise)

3 points. Consider the case where H1 sends 2Mbps traffic to H2 via one of two paths as in Figure 5, either through R1 with link capacities 2Mbps or through R2 with link capacities 4Mbps. Consider the dynamic routing case where the routing table is updated every 3 minutes using a shortest path (least cost) algorithm. When the routing table is updated, the link weight at a link is computed by the following equation:

$$\text{weight} = \frac{\text{1Mbps} + \text{average throughput over the past 3 minutes}}{\text{capacity}}.$$

Assume that at $t = 0$, the routing table is updated, and at $t = 1$ms, H1 starts sending traffic to H2. Give the traffic throughput through Routers R1 and R2 at $t = 1, 4, 7, \ldots$ minutes.

3.5 Routing on a continuum of nodes (optional; 0 point)

Consider the network given in Figure 6. Each point represents a router, connected to its neighbors via links of capacity 1. The links form the ring. Lable the routers by $x \in [0, 1)$, and give router 0 two labels: 0 and 1. Assume that all traffic have the same destination: router 0. Let $r(x)$ denote the traffic arrival rate at router x for $x \in [0, 1)$, and assume $r(x) = 2x$.

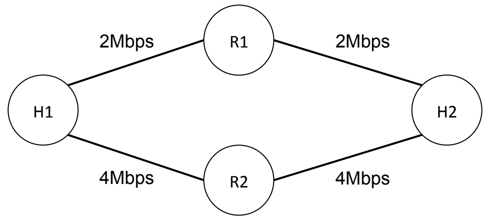

Figure 5: H1 sends 2Mbps traffic to H2 via one of two paths with capacities 2Mbps or 4Mbps.

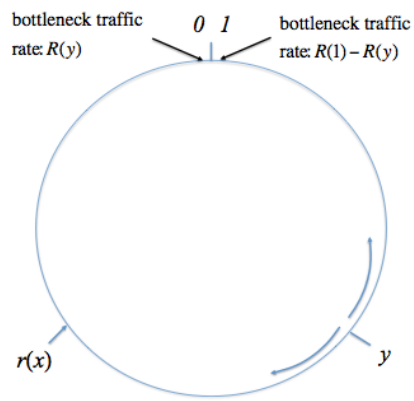

Figure 6: Network topology for problem 3.5.

(a) (Static Routing). Consider the following static routing strategy: pick a $y \in (0,1)$, let every router $x \in (0,y)$ forwards packets clockwise, and every router $x \in (y,1)$ forwards packets counter-clockwise.

- What is the traffic throughput $f^-(x,y)$ at *link* x for $x \in (0,y)$, and what is the traffic throughput $f^+(x,y)$ at link x for $x \in (y,1)$?
- Use the expression of the queueing delay for the M/M/1 queue. What is the queueing delay $d_s^-(x,y)$ at link x for $x \in (0,y)$, and what is the queueing delay $d_s^+(x,y)$ at link x for $x \in (y,1)$? Let $d_s^-(y) := \sup_{x\in(0,y)} d_s^-(x,y)$ denote the maximum queueing delay over links $x \in (0,y)$, i.e., over the links that forward packets clockwise. And let $d_s^+(y) := \sup_{x\in(y,1)} d_s^+(x,y)$ denote the maximum queueing delay over links $x \in (y,1)$, i.e., over the links that forward packets counter-clockwise. What is $d_s^+(y)$ and $d_s^-(y)$?
- Assume that the propagation delay $d_i^-(x)$ from x to 0 (clockwise) is x, and that the propagation delay $d_i^+(x)$ from x to 1 (counter-clockwise) is $1-x$. Each router x has two paths, clockwise or counter-clockwise, to forward packets to the destination—router 0(1). Label the clockwise path by superscript - and the counter-clockwise path by superscript +, and define costs

$$\begin{aligned} D^+(x,y) &:= d_s^+(y) + d_i^+(x), \\ D^-(x,y) &:= d_s^-(y) + d_i^-(x) \end{aligned}$$

  for the two paths. For what values of x is $D^+(x,y)$ equal to $D^-(x,y)$?
- Let $x(y)$ denote the x where $D^+(x,y) = D^-(x,y)$. Use matlab (or other tools) to draw $x(y)$ as y increases from 0 to 1, when does the line intersect $z(y) = y$?

- Let y^* denote the $y \in (0,1)$ where $x(y)$ intersects $z(y)$. Show that

$$\begin{aligned} 0 < x < y \quad &\Rightarrow \quad D^-(x,y) < D^+(x,y), \\ y < x < 1 \quad &\Rightarrow \quad D^-(x,y) > D^+(x,y) \end{aligned}$$

  when $y = y^*$. That is, when $x < y^*$, the left path has smaller cost, and the right path has bigger cost. This is considered "stationary." Give an interpretation of why this is called "stationary".

(b) (Dynamic Routing). Let's extend (a) to the dynamic routing case where routing y is updated over time. Let y^k denote the routing strategy at time $k = 0, 1, 2\ldots$ and assume $y^{k+1} = x(y^k)$ for $k = 0, 1, 2, \ldots$ For what initial values of y_0 does the sequence $\{y_k\}_{y\geq 0}$ converge?

3.6 Forward error correction code

The forward error correction code discussed in the lecture (and textbook) can be represented in the matrix form as in Figure 7, i.e., $C = PA$, where C is a $k \times m$ matrix that represents m encoded packets

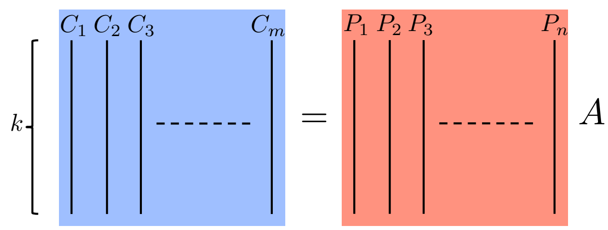

Figure 7: Matrix representation of forward error correction.

each of k bits (the m columns of C), P is a $k \times n$ matrix that represents n original packets each of k bits (the n column of P), and A is an $n \times m$ 0-1 matrix that represents the coding $A_{ij} \in \{0, 1\}$. For example, the jth column of C is:

$$C_j = \sum_{i=1}^{n} P_i A_{ij}$$

for $j = 1, \ldots, m$. Here the summation is elementwise XOR.

Let $A = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$

(a) **3 points.** If packets $C_1 = [1\ 0\ 0\ 1]^T$, $C_2 = [0\ 0\ 1\ 1]^T$, $C_5 = [0\ 0\ 1\ 0]^T$, $C_6 = [1\ 1\ 1\ 1]^T$ are received, find the original packets P_1, P_2, P_3, P_4.

(b) **3 points.** If another set of packets $C_1 = [1\ 0\ 0\ 1]^T$, $C_3 = [0\ 0\ 1\ 0]^T$, $C_4 = [1\ 0\ 1\ 0]^T$, $C_6 = [0\ 1\ 1\ 0]^T$ are received, find the original packets P_1, P_2, P_3, P_4.

3.7 Network coding, W&P, P5.4

Consider a wireless network with nodes X and Y exchanging packets via an access point Z. For simplicity, we assume that there are no link-layer acknowledgments. Suppose that X sends packets to Y at rate 2R packets/sec and Y sends packets to X at rate R packets/sec; all the packets are of the maximum size allowed. The access point uses network coding. That is, whenever it can, it sends the "exclusive or" of a packet from X and a packet from Y instead of sending the two packets separately.

(a) **2 points.** What is the total rate of packet transmissions by the three nodes without network coding?

(b) **2 points.** What is the total rate of packet transmissions by the three nodes with network coding?