CSEC101b

1/27/05

N people to fund a project

- Examples
  - Set up a web-business
  - Create a shared facility: lab, computer, router,..
- The model
- I = 1, …, N
- $u^i(y) - p^i$, u concave, $y \geq 0$
- $\sum p^i = C(y)$, C convex

What is Optimal?

- Max $\sum u^i (y) - p^i$

  Subject to $\sum p^i \geq C(y)$
- Or max $(\sum u^i (y)) - C(y)$
- FOC: $du^i(y^*) /dy = dC(y^*) /dy$
- SOC: concavity and convexity
- p^i can be anything such that $\sum p^i \geq C(y)$

Example

- $u^i(y) = a^i \ln (y)$
- $C(y) = Ky$
- Optimal?
- $\sum (a^i/y) = K$ or
- $y^* = (\sum a^i)/K$

Raise $

- Each i contributes c^i and then $y = (\sum c^i)/K$
- What is the Nash Equilibrium?
- Best reply?
- Max $u^i(\sum c^i/K) - c^i$ implies

  $$[du^i(\sum c^i /K)/dc^i][1/K]= 1$$

- Let y^{*i} solve $du^i(y)/dy = K$
- Then the best reply is $c^i = y^{*i} - \sum_{-i} c^k$

The Nash Equilibrium

- Suppose $c^1 > c^i$ for all $j \neq 1$.
- The Nash equilibrium is

  $c^j = 0$ for all $j \neq 1$

  $c^1 = y^{*1}$
- This is not optimal.
- Let's try something else.

Pay what it is worth

- Let each j pay a price of q^j per unit of y.
- If we set the q^j such that

$$q^j = [du^j(y^*)/dy] = a^j/y^*$$

and $\sum q^j = K$

Then <u>if they take prices as given</u> each j will want y to be chosen such that

$$a^j/y = q^j \text{ or } a^j/y = a^j/y^* \text{ or } y = y^*.$$

And since $\sum q^j = \sum du^j(y^*)/dy = K$, this is optimal.

- The prices q are called <u>Lindahl equilibrium prices</u>.

What process do we use?

- How do we compute the prices?
  - Suppose the fund-raiser knows the form of the utility functions but does not know a.
  - Ask for a^j and then let $q^j = a^j / y^* = Ka^j / (\sum a^k)$
- Note: This is like central planning.
  - If we know a, we can compute y^* and payments $q^j\ y^*$ for each j.
- Will everyone want to tell us what their a^j is?
  - If they were a computer they would, but……

Revelation of information?

- Person j knows the process and knows that if they say m and the others say m, then person j will get

  $$a^j \ln(\sum m^k/K) - [K m^j / (\sum m^k)] (\sum m^k/K)$$

- Maximizing this implies that

  $$[a^j/(\sum m^k/K)](1/K) - 1 = 0$$

- Or $a^j = (\sum m^k)$ for all j
- This is impossible!

The Nash Equilibrium

- So an interior equilibrium does not exist.
- As before $m^{*k} = 0$ for all k but 1 and
  $m^{*1} = a^{1}$
- This is not good.
- Is there anyway we can get every k to tell us their true value of a?

Change the game

- Varian changed the game tree.
- Let's see what happens if we change the payoff functions.
- We do that by changing the payment rules.
- Let each j announce their "parameter" m^j.
- $y^* = \sum m^k/K$
- $T^i(m) = m^i - \sum_{-j} a^j \ln(\sum m^k / \sum_{-j} m^k)$
- Note that $T^i = 0$ if $m^i = 0$

What is the Nash Equilibrium?

- Best reply? Maximize

$a^i \ln (\sum m^k/K) - [m^i - \sum_{-j} m^j \ln (\sum m^k / \sum_{-j} m^k)]$

- FOC
- $a^i / (\sum m^k) - 1 + (\sum_{-i} m^j)/(\sum m^k) = 0$
- Or $a^i + (\sum_{-i} m^j) = (\sum m^k)$
- Or $a^i = m^i$!
- "Truth is a (weakly) dominant strategy."

Generalization: Vickrey-Groves-Clarke

- Payoffs: $u^i (y ; a^i) - p^i$
- Ask for m^i (hoping it is $= a^i$)
- Let $y^*(m)$ maximize $\sum [u^i (y;m^i) -(1/N)Ky]$
- Let $T^i(m) = (K/N)y(m)$

  $- \sum_{-i} [u^k (y(m) ; m^k)-(1/N)Ky(m)]$
  $+ \max \sum_{-i} [u^k (y; m^k)-(1/N)Ky]$

Proof of Incentive Compatibility

- j will want y to maximize

  $u^j(y, a^j) - \{(K/N)y$

  $- \sum_{-i} [u^k (y; m^k)-(1/N)Ky]$

  $+ \max \sum_{-i} [u^k (y; m^k)-(1/N)Ky]\}$

Or $u^j(y, a^j)+ [\sum_{-i} u^k (y; m^k)] - Ky + F$

- The algorithm maximizes

  $\sum u^k (y; m^k)] - Ky$

- So $m^j = a^j$

Possible Problems

- Efficiency in resource use.

  $\sum T^j(m) = K\, y(m)$?
- Generally not. $T^j(m) > (1/N)Ky(m)$.
- There are other processes that are not "optimal" in the choice of y but which are efficient in resource use and which are Pareto-superior to VGC.
  – Majority Rule is one.

Majority rule

- $u = a \ln(y) - p$, $C(y)$
- Propose a series of y's until we find a y' such that there is no other y that a majority prefer. Each j pays (K/N)y'.
- Let y^j solve max $a^j \ln y - (K/N)\, y$.
- What is the majority rule equilibrium?

Median Voter theorem

- Let y’ be the median $\{y^1,\ldots,y^N\}$.
- Theorem: If the u are concave, then y’ is the majority rule equilibrium. (If N is odd and u are strictly concave, it is unique.)
- Proof:

Incentives

- A direct mechanism: report a and the mechanism picks the median.
- Theorem: Truth is a dominant strategy
- Revelation Principle
- Corollary:It is dominant strategy to vote your true preferences.

Observation

- There are parameters a for which

$\sum [u^i (y') - p'^I] > [\sum u^i (y^*) - p^{*i}]$

Even though

$\text{Max} \sum u^i (y') - C(y') < \sum u^i (y^*) - C(y^*)$

Because $\sum p'^i = C(y) < \sum p^{*i}$