CS/Ec101b (2005)

Game Theory Module

Possible References, Texts

- Fun and Games: A Text on Game Theory
  - Ken Binmore, Heath 1992
- Game Theory: Analysis of Conflict
  - Roger Myerson, Harvard Press 1991
- Thinking Strategically
  - Avinash Dixit and Barry Nalebuff, Norton 1991
- Games and Decisions
  - Duncan Luce and Howard Raiffa, Wiley 1957
- Evolution and the Theory of Games
  - John Maynard Smith, Cambridge U Press, 1982

Intro

- Game theory is about strategic interactions
- Should a committee member ever vote for their least favorable candidate?
- Might it be best to flip a coin to decide whether to attack today or tomorrow?
- Should someone contracting to build a new facility consider holding an auction and paying the second lowest price?

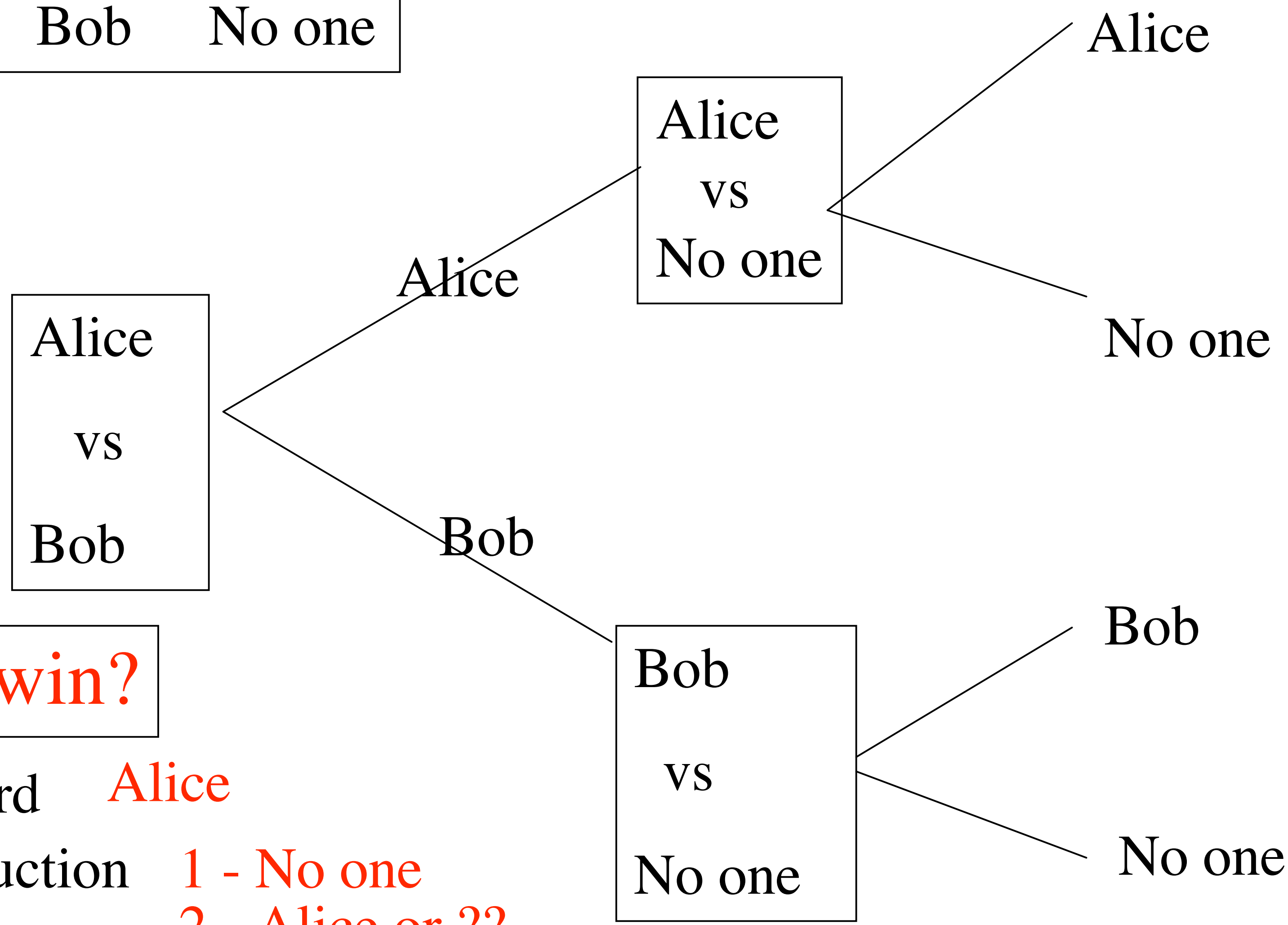
Strategic Voting

- 3 roommates Kevin, Lois, and Minh use majority rule voting to decide whether to add another roommate or not.
- Alice is the first proposed. But then someone mentions Bob.
- They vote first on whether to replace Alice with Bob (the amendment) and then whether to add the winner of that or not.

Kevin	Lois	Minh
Alice	No one	Bob
No one	Alice	Alice
Bob	Bob	No one

Who should win?

What is the process?



Who does win?

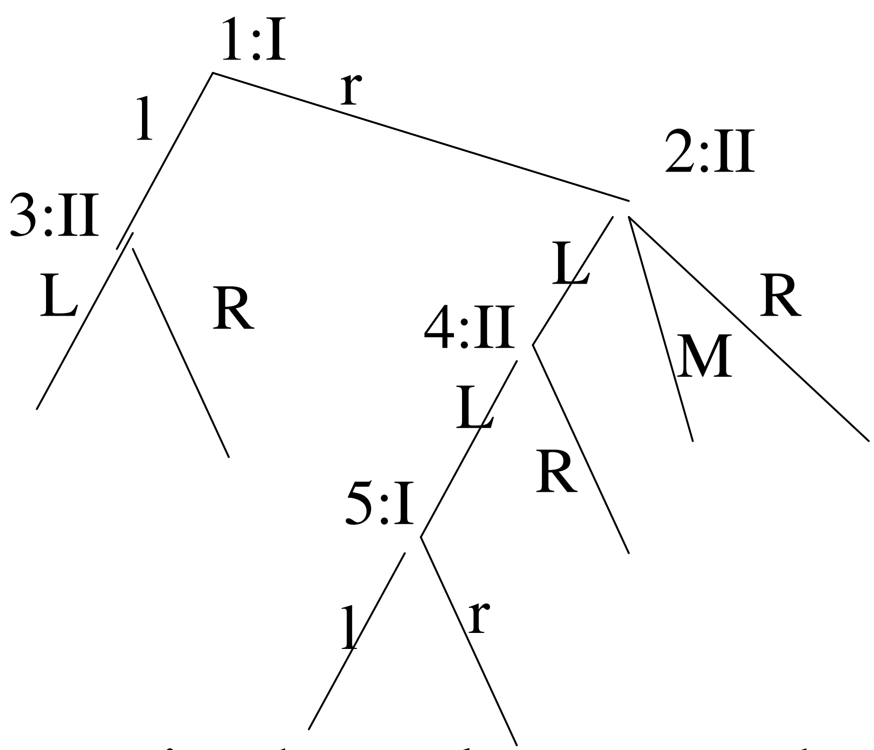
Straight-forward Alice

Backward-induction 1 - No one
2 - Alice or ??

The Rules of the Game

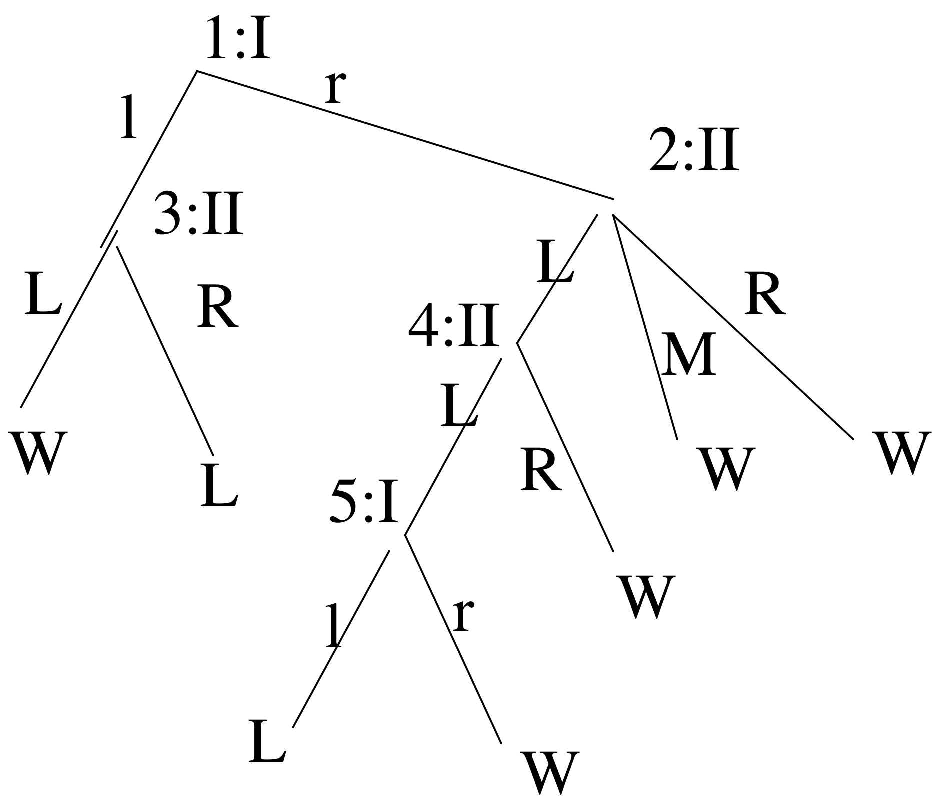
- Who can do What and When?
- Who gets How much when the game is over?
- When - Extensive Form
  - A game tree: a connected graph (nodes and links) with no cycles

Who and What?

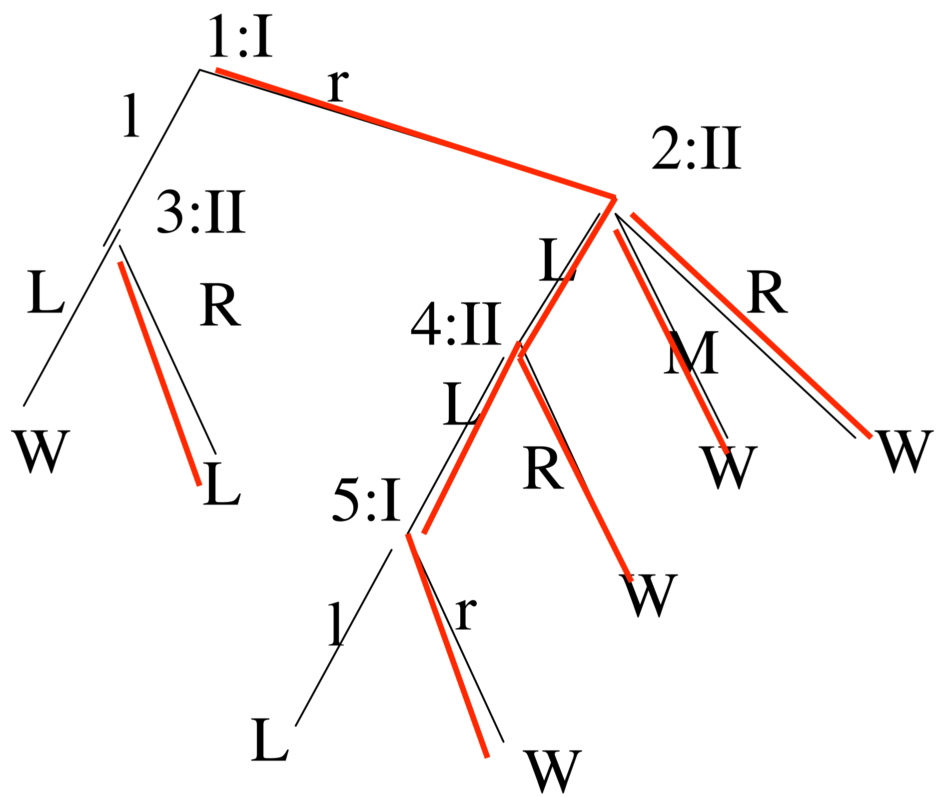


- Strategy: an action by a player at each node to which they are assigned.
  - Player 1: at 1 and 5, (l,l),(l, r),….
  - Player 2: at 2,3 and 4, (x, y, z) …

How Much?



Playing to Win: Backward Induction



The "equilibrium" strategies are (r, r) and (R, x, x).

Strategic Form

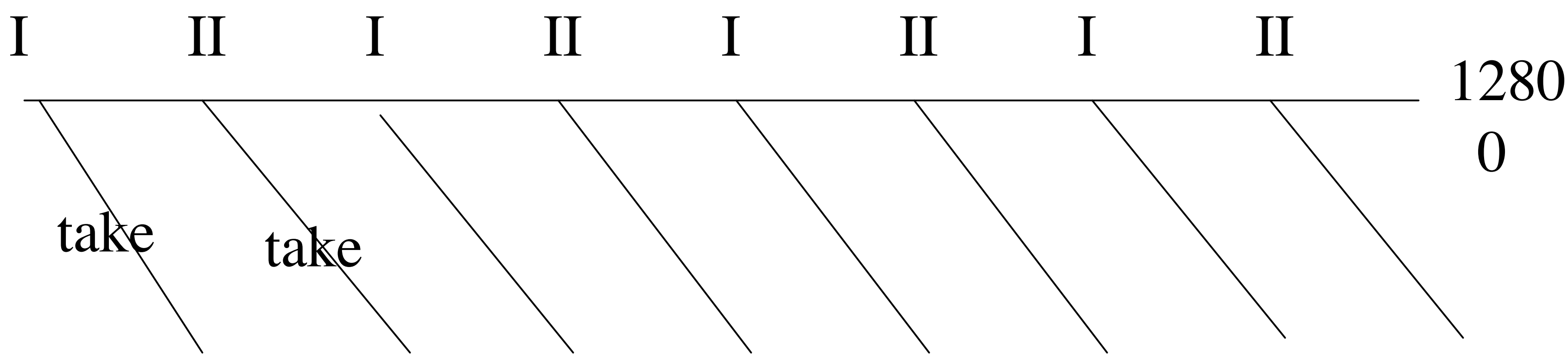
	L L L	L L R	L M L	L M R	L R L	L R R	R L L	R L R	R M L	R M R	R R L	R R R
ll	W	W	W	W	W	W	L	L	L	L	L	L
lr	W	W	W	W	W	W	L	L	L	L	L	L
rl	L	W	W	W	W	W	L	W	W	W	W	W
rr	W	W	W	W	W	W	W	W	W	W	W	W

(Weak) Best Reply Strategy: s' is a Best Reply to t if $\prod(s', t) \geq \prod(s, t)$ for all s.

(Weakly) Dominant Strategy : $\prod(s', t) \geq \prod(s, t)$ for all s and t.

Note that rr is a (weakly) Dominant Strategy for I and RLL is a (weakly) Dominant Strategy for II.

Behavioral Example: Should you follow the advice of a game theorist? (Aumann)



Player 1	10	0	40	0	160	0	640	0
Player 2	0	20	0	80	0	320	0	1280

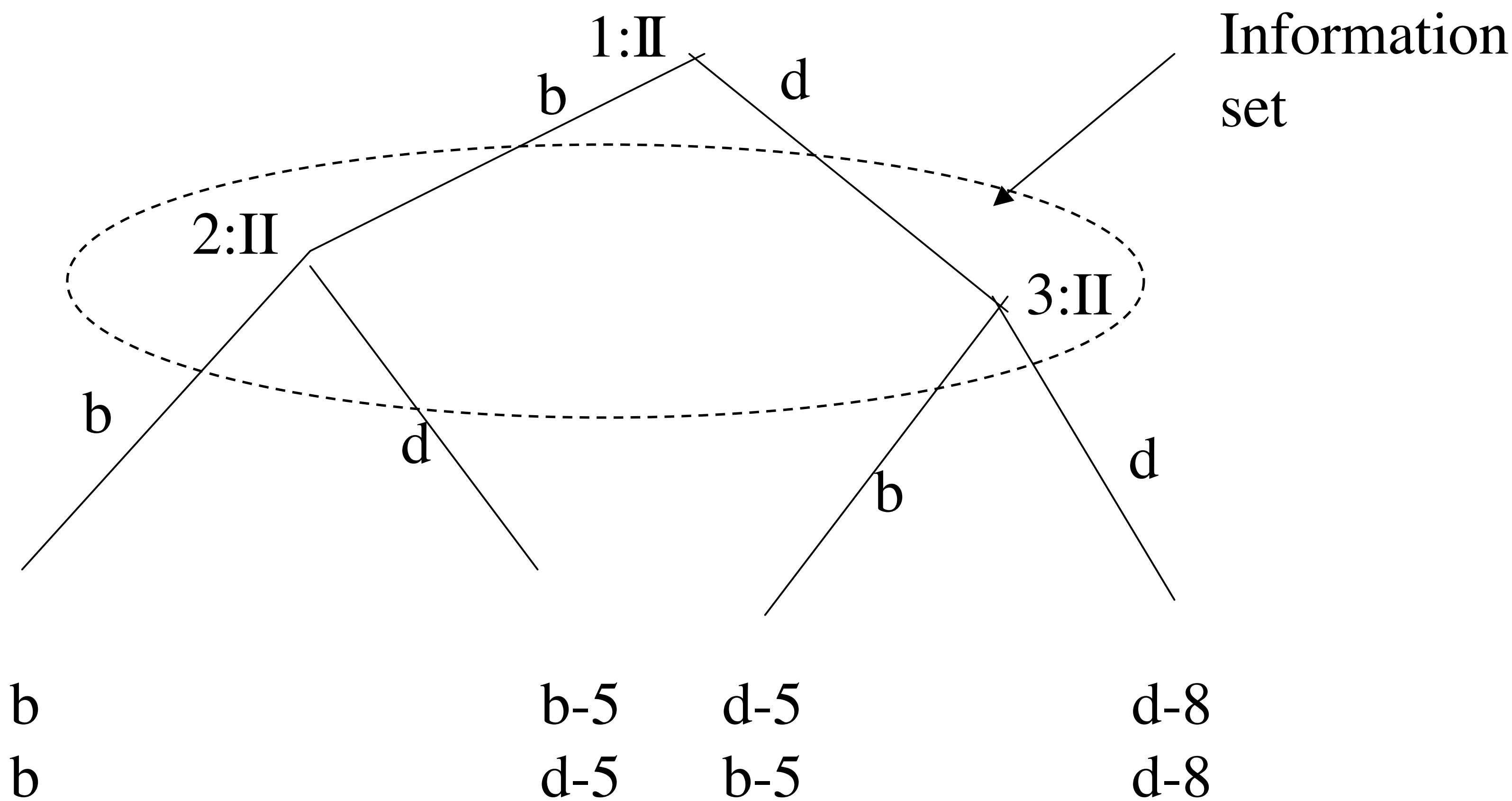
The Theorist's Advice: What does backward induction imply?
What would you do, if you were II, at your first chance? If you were I?

In the lab most people pass the first time. (McKelvey and Palfrey)

Simultaneous Move Games

- Let’s consider a Dilemma
- Do you drive(D) to school or do you ride a bike(C)?
  - Driving creates pollution which no one likes
    - $P(0) = 0$, $P(1) = 5$/person, $P(2) = 8$/person
  - Driving yields higher benefits (pollution aside)
    - $U(D) = d$, $U(B) = b$, $d > b + 5$

The Extensive Form



The strategy for II has to be the same at node 2 and 3.

The Normal Form

	b	d
b	b, b	b-5, d-5
d	d-5, b-5	d-8, d-8

Now d is a best replay to b and d is a best replay to d.

So d is a dominant strategy for both!

Why is this a Dilemma? If 5 < d-b < 8, then both would be better off at b. There is a Pareto-Superior action plan.

Not every game has a Dominant Strategy Equilibrium

	L	R
U	10, 2	4, 20
D	3, 7	11, 15

For row player: U is b.r. to L, D is b.r. to R. Which to play?

Look at column player: R is b.r. to U and R is b.r. to D
Note that D and R are b.r. to each other.
(Stable?) (Publish proof?)

Nash Equilibrium: A vector of strategies such that each is a best replay to the others. (D, R) is the Nash Equilibrium. DS⊆NE

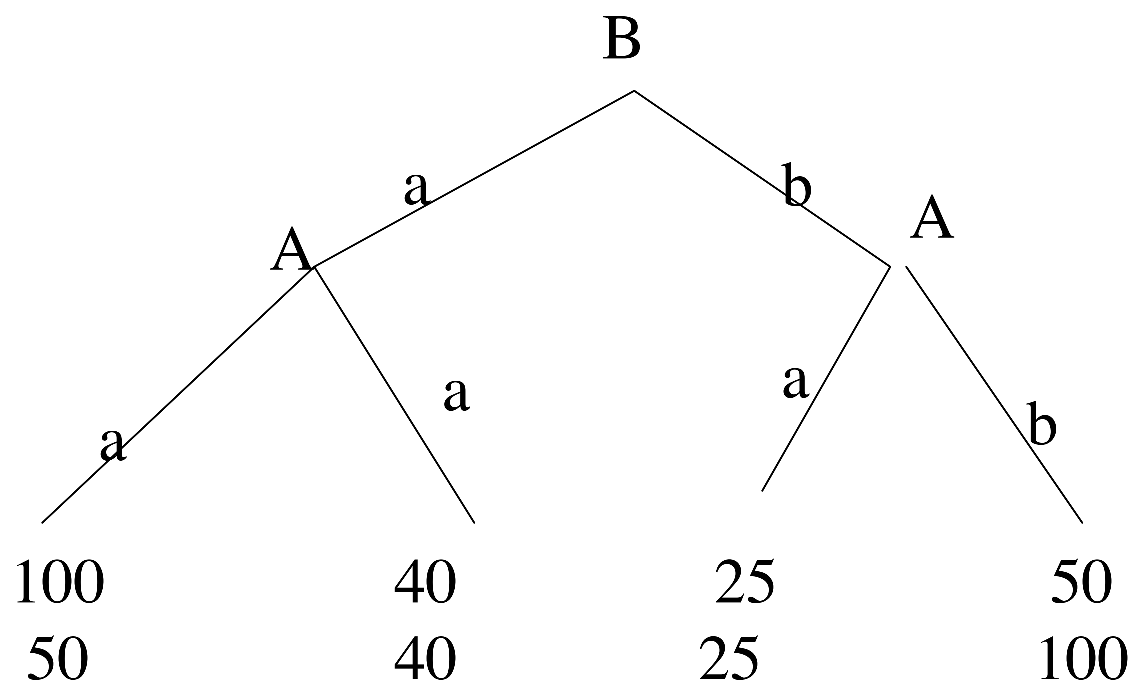
Nash is Not Enough

- Consider the Boeing/Airbus game.
  - They can each choose design a or b.
  - B prefers a, A prefers b.

B \ A	a	b
a	100, 50	40, 40
b	25, 25	50, 100

There are two Nash Equilibrium: (a,a) and (b,b)

The Extensive Form

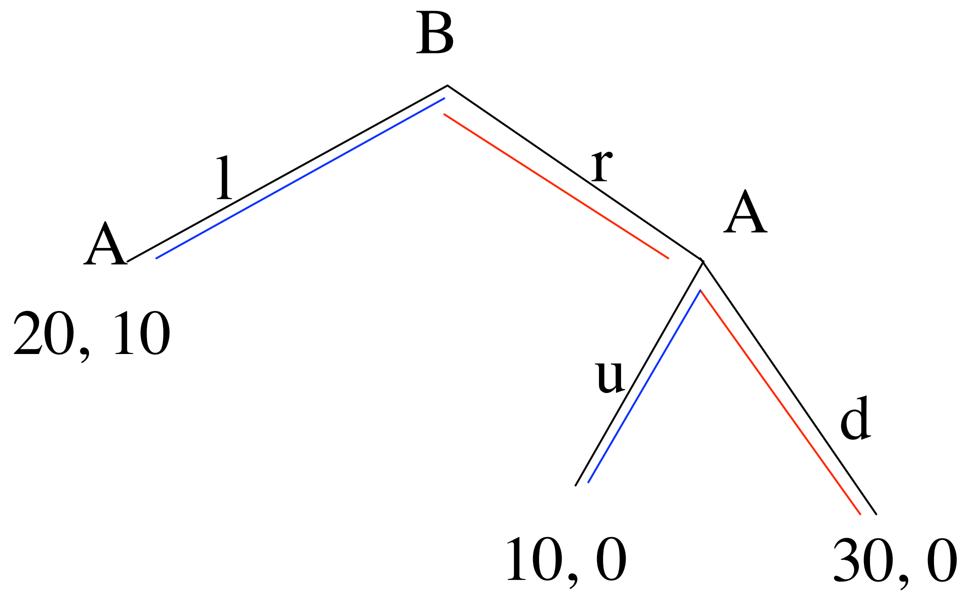


Backward induction implies (a,a).
First Mover advantage in a coordination game

Note: A could try to threaten that they will use b "no matter what" but it is not credible.

(Selten) Subgame Perfect Nash Equilibrium is a Nash Equilibrium in all subgames. $SPE \subset NE$

2 SPE



(r, d) and (l, u) are subgame perfect equilibria.

Can B count on A to pick d in that sub-game? If not then what?
A can gain by seeming to be “irrational”.

There may be NO (pure strategy) Nash Equilibria

- The soccer penalty kick (or tennis serve)
  - Strategies
    - Kicker: Left or Right
    - Goalie: Left or Right
  - Payoffs
    - If both go same way, no goal is scored (-1, 1).
    - If they go different ways, a goal is scored (1, -1).

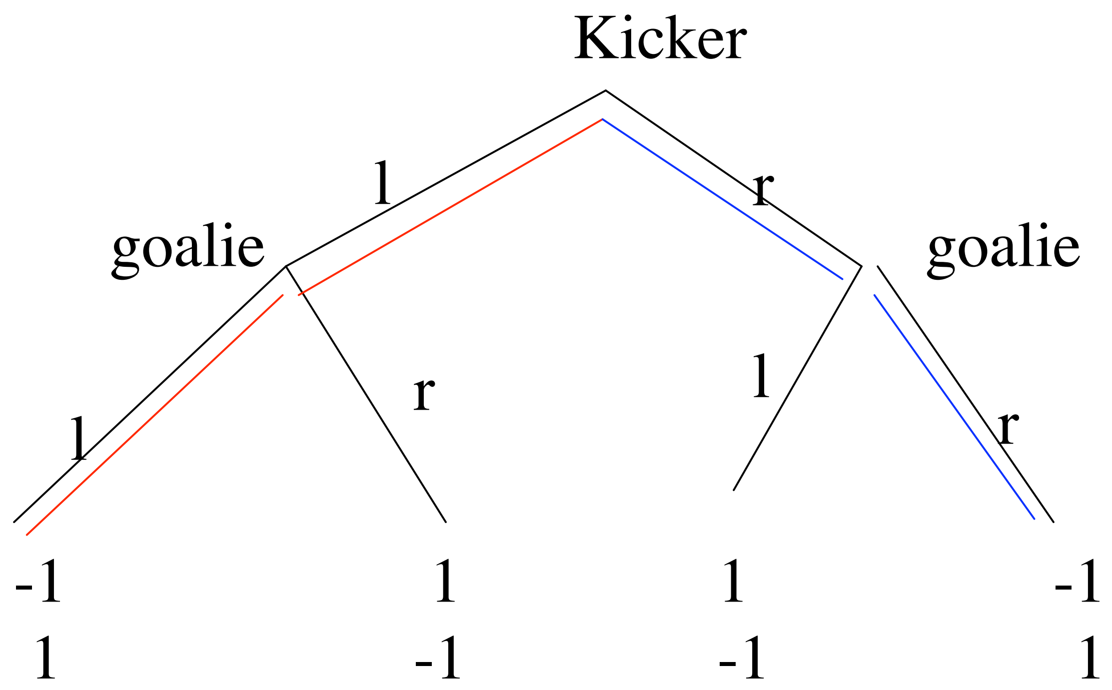
Normal (strategic) Form

K \ G	Left	Right
Left	-1, 1	1, -1
Right	1, -1	-1, 1

KL is best against GR which is best against KR which is best against GL which is best against KL.
There is no Nash Equilibrium here.

This is a Zero-Sum game.

The Extensive Form



There are 2 SPE: (l, l) and (r, r)

In strategic form there is a second mover advantage.

How do I protect against the other player figuring out what I am going to do?

Mixed Strategies

- To protect against the second mover, I can use a Mixed Strategy: I randomize.
- Suppose the kicker plays L with probability k?
- If the goalie plays L they get <u>in expected value</u> k (1) + (1-k) (-1) = 2k - 1.
- If the goalie plays R they get 1-2k.
- The goalie will play L if and only if 2k-1 > 1-2k or k > 1/2.

Mixed Strategies

- The goalie will play L if and only if $2k-1 > 1-2k$ or $k > 1/2$.
- So the kicker gets $1-2k$ if $k > 1/2$ and gets $2k-1$ if $k < 1/2$.
- The best k the kicker can choose is 1/2.
- This makes the goalie indifferent between his possible responses.
- A mixed strategy is a probability density over your space of possible pure strategies.

Finding Equilibrium in Strategic Form

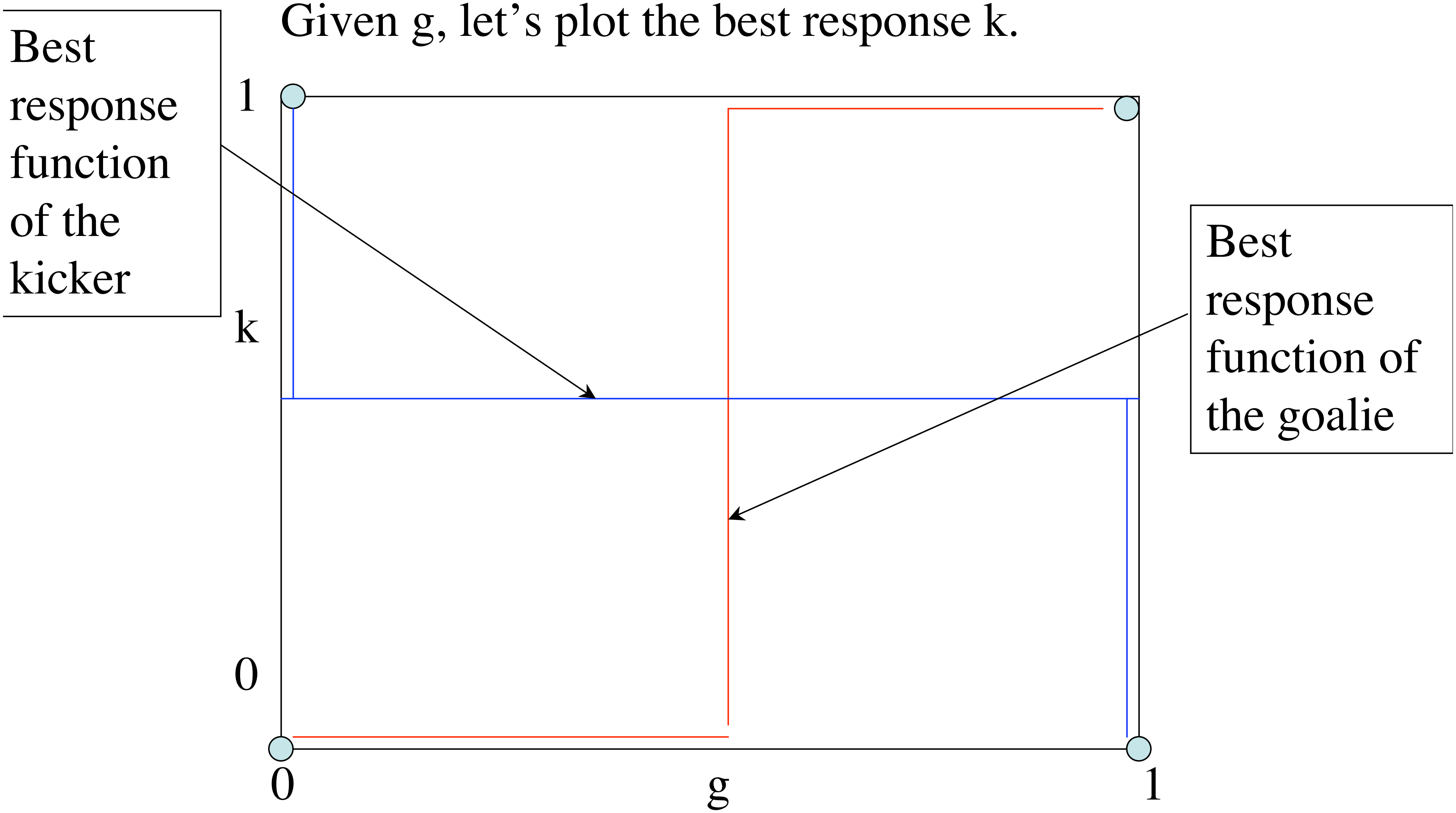
- If the kicker uses a mixed strategy k and the goalie uses a mixed strategy g then
  - The goalie gets

  $g[k\,(1) + (1\text{-}k)\,(\text{-}1)\,] + (1\text{-}g)[k\,(\text{-}1) + (1\text{-}k)\,(1)]$

  $= g\,(2k - 1) + (1\text{-}g)(1\text{-}2k) = 4gk - 2g - 2k + 1.$
  - The kicker gets $1 - 2k - 2g - 4gk$.
- What are the equilibrium values of k and g?

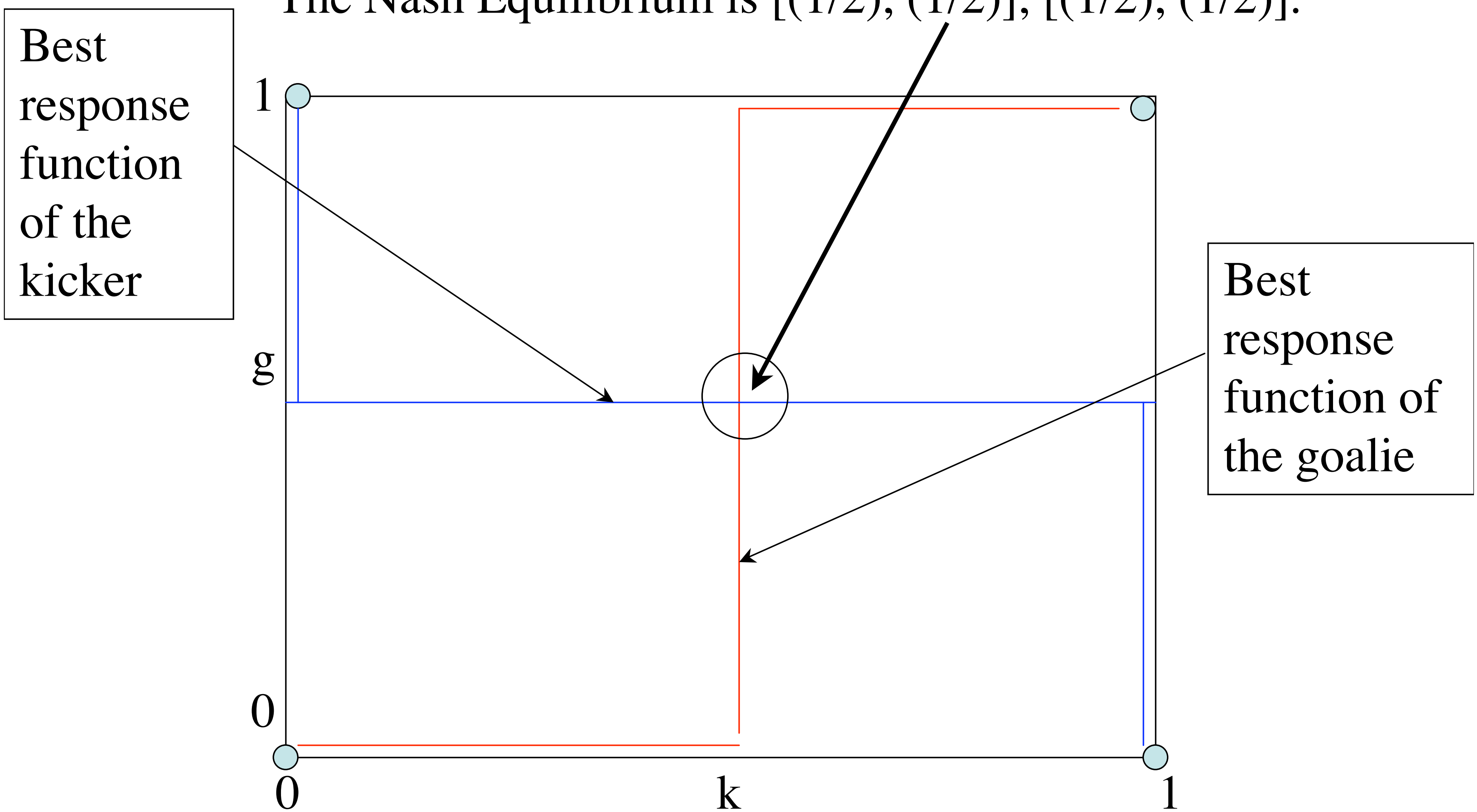
$$U(k,g\text{: goalie}) = 4gk - 2g - 2k + 1$$

Given g, let's plot the best response k.



Now do g given k, where $U(k,g\text{: kicker}) = -4gk+2g+2k-1$

The Nash Equilibrium is [(1/2), (1/2)], [(1/2), (1/2)].



Next

- Theorem: existence of equilibrium
- Rationalizability
- Iterated dominance