CALIFORNIA INSTITUTE OF TECHNOLOGY
Selected Topics in Computer Science and Economics

CS/EC/101b

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1. You are shown three closed rooms. Two of the rooms contain $0 and one of the rooms contains $300,000. (In the actual game, two of the rooms contain goats and the third room contains a Cadillac.) You cannot see into the room and you don't know which room contains what amount. You do, however, know the rules and the amounts in the rooms. You are offered the following games.

   - **Game 1**: You choose one of the three rooms at random and tell the game host (Monty Hall) which room you've chosen. The game host then opens one of the two rooms that you haven't chosen; he opens the room containing $0 and shows you the room. You now open the room other than the room that you chose initially, and you take the prize in the room that you've opened.
   - **Game 2**: You choose one of the three rooms at random and tell the game host which room you've chosen. The game host then opens one of the two rooms that you haven't chosen and shows you the room; he opens the room containing $0. You open the room that you chose initially and take the prize in that room.
   - **Game 3**: You choose one of the three rooms at random but don't tell the game host which room you've chosen. The game host then opens one of the rooms containing $0 and shows you the room. You open a room and take the prize in it.

   Compute the expected value of each of these games. (18 points)

2. The cost of playing the following game is $25.

   The kings and queens (4 kings and 4 queens) from a regular pack of cards are taken out to form an 8–card pack. The 8–card (4 kings and 4 queens) pack is shuffled. A card is drawn from this 8–card pack at random; call this card1. Now there are only 7 cards left in the pack. Then a second card is drawn from the remaining pack at random; call this card2. You can't see the cards that are drawn or the cards in the pack.

   The outcome of the game is as follows: If both the cards drawn (card1 and card2) are kings then you get $100, otherwise you get $0.

   You can bribe the card shuffler to get some information. Different bribes cost different amounts and give you different amounts of information. Here is the list of bribes:

   - **Bribe 1**: Cost $6. Information: You are told (truthfully) if card1 is a king or is not a king.
   - **Bribe 2**: Cost $5. Information: You are told (truthfully) if card2 is a king or is not a king.

- **Bribe 3**: Cost \$4. Information: You are told (truthfully) whether one of the two cards drawn is a king.
- **Bribe 4**: Cost \$3. Information: You are given some information about card1, but the information is "noisy" i.e., imperfect. The information that you are given is incorrect with probability 0.1. If card1 is a king you are told that it is a king with probability 0.9 and you are told that it is a queen with probability 0.1. If card1 is a queen, you are told that it is a king with probability 0.1 and that it is a queen with probability 0.9.
- **Bribe 5**: Cost \$2. Information: You are told truthfully whether one of the two cards is the king of diamonds. If one of the two cards is the king of diamonds you are told that fact, and if neither of the two cards is the king of diamonds, you are told that fact too.
- **Bribe 6**: Cost \$1. Information: You are told truthfully whether one of the two cards is a red king. (A red king is a hearts or diamonds king.)

You can make at most one bribe. (We remind you that bribing violates the Caltech honor code and the ethical standards of all religions, but for the purposes of this problem, and only for the purposes of this problem, bribes are acceptable.) If you bribe you get information from the bribe; based on the information you then decide whether to pay \$25 to play the game, i.e., to bet that both cards drawn are kings.

(a) Assume that your goal is to maximize the expected dollar value. What is the optimal strategy? Prove your result. (30 points)

(b) Write a short paragraph on the value of information. Is more information always better than less information (i.e., "king of diamonds" better than "red king", or is "first card is a king" better than "one of the two cards is a king")? Why or why not? (10 points).

3. An oil wildcatter is prospecting for oil on a ranch that she has purchased in West Texas. Based on the landscape of the region and other information, she believes that the probability of oil on the ranch is 0.01. For the purposes of this problem assume that there are only two states: the ranch has oil or the ranch has no oil. (Of course, in reality, the amount of oil on the ranch could vary from 0 barrles to millions of barrels, and the amount is important. But for this problem ignore amounts and treat the situation as binary: either oil exists or it does not.)

   The wildcatter can get an oil–services company to conduct a seismic test on the ranch for a cost of \$100,000. The seismic test gives her an indication of whether there is oil on the ranch or not. The test gives correct results with probability 0.9. If there is oil on the ranch then the seismic test correctly predicts that there is oil with probability 0.9 (and incorrectly predicts that there is no oil with probability 0.1). If there is no oil on the ranch then the seismic test incorrectly predicts that there is oil with probability 0.1 and correctly predicts that there is no oil with probability 0.9.

   She can get the oil services company to drill for oil, and she can make this decision without conducting the seismic tests, or she can conduct the seismic test and wait for the result of the test before deciding whether to drill or not. Based on experimental results produced by the oil services company she knows that if there is oil on the ranch, the probability that the drill will produce oil (let's call this situation a "gusher") is 0.95 and hence the probability that the

drill won't produce oil (let's call this situation "a dry hole") is 0.05. If there is no oil on the ranch then the probability that the drill will produce oil is 0.

Assume that the outcomes of the tests, given the state, are independent. For example, given that there is oil, the outcomes of the seismic test and the final drilling are independent of each other. For example, suppose there really is oil on the ranch, then given that fact (oil on the ranch) the probability that a seismic test is positive and a drilling produces a gusher is $0.9 \cdot 0.95$.

The cost of drilling is \$1,000,000. The value of a gusher is \$20,000,000, and the value of a dry hole is \$0.

(a) What is the optimal strategy if the wildcatter's goal is to maximize expected dollar value? Prove your answer. (30 points)

(b) This question is personal. It deals with you as an individual and there is no right answer. Suppose you were the wildcatter and you had to design a strategy for yourself. You can go to the bank and borrow money at 12% interest to conduct the test and drill. What would you do? Is your answer for part 2 different from that of part 1? If so, why? Are you irrational? If not, why not? (12 points)