CALIFORNIA INSTITUTE OF TECHNOLOGY
Selected Topics in Computer Science and Economics

CS/EC/101b

K. Mani Chandy, John Ledyard **Homework Set #3** Issued: 31 Jan 05
Winter 2005 Due: **BEFORE CLASS** 8 Feb 05

All students should complete the following problems:

1. In a game called Blackball, a committee of 3 club members (I, II, III) has to select one from a list of four candidates (A, B, C, D) as a new member of the club. Each committee member is allowed to blackball (veto) one candidate. This right is exercised in rotation, beginning with player I and ending with player III. The players preferences satisfy: (where $A \succ_1 B$ means I prefers A to B):

$$\begin{aligned} A &\succ {}_1B \succ_1 C \succ_1 D \\ B &\succ {}_2C \succ_2 D \succ_2 A \\ C &\succ {}_3D \succ_3 A \succ_3 B \end{aligned}$$

   (a) What is the tree for this game?

   (b) How many pure strategies does each player have?

   (c) Who gets elected to the club if a subgame perfect equlibrium is used?

   (d) Find at least one Nash equilibrium that is not subgame perfect.

2. The Chain–store Game is often used to illustrate the logic of "entry deterence". Player I is an incumbant monopolist in an industry who makes \$5m if left alone to enjoy his priviledged position undisturbed. Player II is a firm that could enter the industry but earns \$1m if it chooses not to enter. If the potential entrant decides to enter, then the monopolist can do one of two things: he can fight by flooding the market with his product so as to force down the price, or he can acquiesce and split the market with the entrant. A fight is damaging to both players. They then make only \$0m each. If they split the market, each will make \$2m.

   (a) What is the game tree for the Chain–store Game?

   (b) The incumbant monopolist will threaten the potential entrant that he will fight if she disregards his warning to keep out of the industry. Why will she not find his threat credible?

   (c) What will happen if both players act rationally (and this is common knowledge)?

3. Suppose the incumbant monopolist in 3 can decide before anything else happens, to make an irreversible investment in extra capacity. This will involve a loss of \$2m if he makes no use of the capacity, and the only time the capacity will be used is if he fights the entrant. He will then make \$1m (inclusive of the cost of the extra capacity) instead of \$0m because the existence of the capacity will make it cheaper for him to flood the market. Player II's payoffs remain the same.

(a) Draw the new game tree illustrating the new situation.

(b) What is the unique subgame perfect equilibrium?

(c) Someone who knows no game theory might say it is "irrational" to invest in extra capacity that you do not believe you will ever use. How might a game theorist reply?

4. (choose 2/3 of average) Each of 3 people simultaneously announce an integer from 1 to K. If the three integers are different, the person closest to 2/3 of the average wins 1\$. If two or more of the integers are the same, \$1 is split evenly among those closest to 2/3 of the average.

   (a) Is there an integer k such that the strategy profile (k, k, k) is a Nash equilibrium of this game?

   (b) Is any other action profile a Nash equilibrium?

   (c) Find all rationalizable strategies for each player in this game.