

CALIFORNIA INSTITUTE OF TECHNOLOGY
Selected Topics in Computer Science and Economics

CS/EC/101b

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Winter 2005

Homework Set #2 Issued:

19 Jan 05

Due: **BEFORE CLASS** 25 Jan 05

All students should complete the following problems:

1. Formulate the following problem as an optimization problem. New students are assigned to houses during “rotation week”. Each student is assigned to exactly one house. Associated with each house h is a given number of vacancies v_h in that house. Let n be the number of new students. The total number of vacancies over all the houses is n .

After the interviews between new students and house members, each new student and the house president together assign a happiness index for that student joining that house. If the k -th new student joins house h , then the happiness (*utility*) for that student in that house is some given value $c_{k,h}$.

- (a) Attempt to formulate the problem in the standard canonical form:

$$\max \mathbf{c}^T \cdot \mathbf{x} \text{ subject to } \mathbf{A} \cdot \mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0}. \text{ (20 points).}$$

- (b) Do you see any problems with your formulation? If so, what are they? (5 points).

2. Let

- $\mathbf{c}^T = (9, 7, 6, 9)$
- $\mathbf{A} = \begin{pmatrix} 4 & 2 & 1 & 3 \\ 1 & 1 & 3 & 1 \end{pmatrix}$
- $\mathbf{b} = (b_1, b_2) = (8, 9)$

Solve the optimization problem: find $z = \max \mathbf{c}^T \cdot \mathbf{x}$ subject to $\mathbf{A} \cdot \mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0}$, using the *pivoting method of simplex*, where $\mathbf{b} \geq \mathbf{0}$. (35 points).

3. (a) Solve the same problem using Matlab with the following settings:

- $b_1 = 0, 2, 4, 6, 8, 10, 12$ and $b_2 = 9$

Turn in a plot of the objective value z as a function of b_1 . (30 points).

- (b) Comment on your results. What is the dual variable, the “price” of the resource b_1 at each point in the graph in terms of the graph? How does the price of resource b_1 change as b_1 is increased? What is the price of resource b_1 when b_1 is made arbitrarily large? (10 points).

1 Matlab Tutorial

1.1 How to get started

On window platforms you get started with MATLAB by double-clicking the MATLAB shortcut icon on your Windows desktop. On UNIX platforms, you can start MATLAB by typing `matlab` at the operating system prompt. When you start MATLAB, the MATLAB desktop will appear. Here are some of the main features you need to know:

- To create a new file, select **File** → **New** → **M-File**. The name of the Matlab file must be the same as the name of the Matlab function which you have created.
- Use the command window to run functions and enter variables. For instance, suppose you have created the Matlab function `Average(x,y)` which computes the average of two numbers and you want to execute it. The first step is to set the value for `x` and `y`. Suppose that you want to compute the average of 4 and 6. Then you type `x = 4` and press enter in the command window. Subsequently you type `y = 6` and press enter. Finally you type `Average(x,y)` and the value 5 will be printed on the command window.
- Use the Help browser to search for and view documentation. The Help browser is an HTML viewer integrated into the MATLAB desktop. To open the Help browser, just click the help button in the desktop toolbar. The Help browser consists of two panes, the Help Navigator, which you use to find information, and the display pane, where you view the information. If you are completely unfamiliar with Matlab, it is a good idea to view the Matlab demo. You can do this by selecting **Help** → **Demos**. The demo will illustrate the main Matlab programming features.
- The Matlab function `plot` can be used to plot the obtained results. The basic format of the function is to enter the following command in the Matlab command window or into a m-file: `plot(X,Y)`, where `X` and `Y` are two vectors having the same number `n` of components. This command will plot on a cartesian plane a set of `n` points $Z(i) = (X(i), Y(i))$, where the first component of $Z(i)$ is the `i`-th component of vector `X` and the second component of $Z(i)$ is the `i`-th component of vector `Y`.

1.2 The function `linprog`

You may want to use the Matlab function `linprog` to solve problem 3. A short example on how to use `linprog` has been posted on the web page of the course. For further information you can enter `help linprog` from the command window or use the help browser as described before. Notice that `linprog` is designed to solve linear programming problems where the objective is to minimize a linear objective function subject to a set of linear constraints. In problem 3 you are asked to find the maximum of the objective function subject to a set of linear constraints. After observing that the maximum of a function f over a set of constraints is equivalent to the minimum of $-f$ over the same set of constraints, we can use the matlab function `linprog` to compute it. However, remember to change the sign of the returned value when you plot your results.