CALIFORNIA INSTITUTE OF TECHNOLOGY
Selected Topics in Computer Science and Economics

CS/Ec 101b

K. Mani Chandy, John Ledyard — **Homework Set #1** — Issued: 7 January 2005
Winter 2005 — Due: 17 January 2005

All students should complete the following problems:

1. Read the slides, which are available from the course schedule page.

2. Let g be a convex function from a tuple space to the real space. Consider the set:

$$H = \{x | g(x) \leq d\}$$

for any number d. Assume H is non–empty.

   (a) Show that H is a convex set or give a counter example. (24 pts)

   (b) Show that H is a closed set or give a counter example. (24 pts)

3. Consider the optimization problem:

$$z = \max f(x)$$
$$\text{subject to: } \forall j, g_j(x) \leq b_j$$

where f is a concave function, and g_j are convex functions for $j = 1, 2, \ldots, m$. Consider the case where this problem is solved repeatedly for different vectors b. We can then study the value of z for different values of b.

Let $z(b)$ be the function defined as:

$$z(b) = \max f(x)$$
$$\text{subject to: } \forall j, g_j(x) \leq b_j$$

   (a) Is function z concave, convex, none of these? Prove your result. (30 pts)

   (b) Is function z always linear? You do not need to give a proof (5 pts) (just answer yes or no).

   (c) Is function z monotone non–decreasing, monotone non–increasing, none of them? A function z is monotone non decreasing if for any $b \geq q$, we have that $f(b) \geq f(q)$. When we say $b \geq q$, we mean that for each i^{th} component $b_i \geq q_i$. Prove your result. (15 pts)

4. Send an email to cs101b@cs.caltech.edu including your name, your email address, and whether or not you are taking this course for credit. You are also welcome to ask any question concerning the course (lectures, homework, etc. . .) using the same email address. (2 pts)

All work should be turned in to the box outside Jorgensen 62 by 23:59:59 Monday, 17 January 2005.