# Proofs and Experiments in Scalable, Near-Optimal Search by Multiple Robots

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## Introduction

• Locating a non-adversarial target using multiple robotic searchers







## Introduction

- Multi-robot Efficient Search Path Planning (MESPP)
  - Target non-adversarial
  - Try to maximize the probability of finding target over a time interval
  - The environment is known to the searcher

## **Difficulties**

- Path planning for multiple robot is NP-hard
- Problem grow exponentially
- Single robot also difficult in large environments

## Solutions

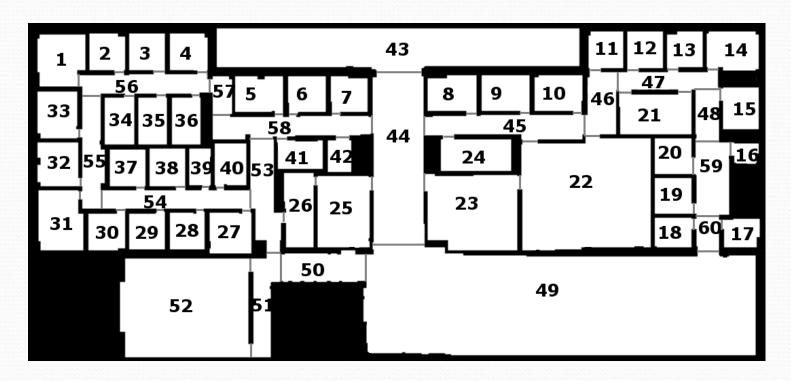
- Approximate algorithm
- Implicit coordination sequential allocation
- Running time linear
- Good bounds because the problem is submodular

# Problem Setup

- Movement model of target: moves according to a Markov chain
- Target's movement model is known to searchers
- Searcher knows its own position and it has knowledge of the target's position at a time *t* in the form of a belief distribution over all possible locations

# **Example Environment**

 Environment: discretized to connected convex regions – undirected graph



### **Reward Function**

- G(N,E), expanded to G'(N',E) for all time t
- At any time *t*, a searcher exists on vertex *s*(*t*) in *N*
- A target also exists on this graph on vertex e(t) in N
- Reward *R* gained when s(t) = e(t), discounted by  $\gamma^t$
- Reward function *F* for search path *A* iterating through all possible target path *Y*:

$$F(A) = \sum_{Y \in \psi} P(Y) F_Y(A)$$

# **Proving Submodularity**

- Show that  $F(A) = \sum_{Y \in \psi} P(Y) F_Y(A)$  is submodular
- Reminder of submodularity:

$$A \subset B, s \notin B \to F(A+s) - F(A) \ge F(B+s) - F(B)$$

- Why is  $F_Y(A)$  given Y submodular?
- Each  $F_Y(A)$  given Y is submodular, so the weighted sum of submodular function is also submodular
- Since target moves in a Markov chain, computing the sum is efficient

## Algorithm: Sequential Allocation

Algorithm for MESPP using ESPP:

#### Algorithm 1 Sequential allocation MESPP algorithm

Input: Multi-agent efficient search problem

 $% V \subset N'$  is the set of nodes visited by searchers

$$V \leftarrow \emptyset$$

for all searchers k do

%  $A_k \subset N'$  is a feasible path for searcher k

% Finding this  $arg \max solves$  the ESPP for searcher k

$$A_k \leftarrow \operatorname{arg\,max}_{A_k} F(V \cup A_k)$$

$$V \leftarrow V \cup A_k$$

#### end for

Return  $A_k$  for all searchers k

# Algorithm: Finite Horizon Planning

Algorithm 2 Finite-horizon path enumeration for ESPP

Input: Single-agent efficient search problem

for All paths A to horizon d do

Calculate F(A)

end for

Return  $A \leftarrow \arg \max_A F(A)$ 

# **Approximation Guarantee**

• Finite horizon bounds:

$$F(A^{FH}) \ge F(A^{OPT}) - \epsilon$$
  $\epsilon = R\gamma^{d+1}$ 

- Why?
- Approximation guarantee of MESPP is  $(1+\kappa)$  where  $\kappa$  is the approximation guarantee of the ESPP problem
- MESPP using finite horizon planning:

$$F(A_1^{FH},...,A_1^{FH}) \ge \frac{F(A_1^{FH},...,A_1^{FH}) - \epsilon}{2}$$

# Measurement Incorporation

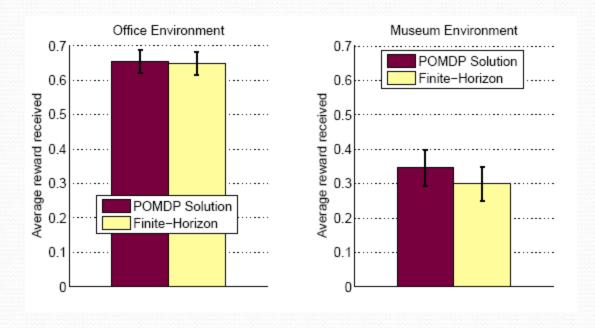
- MESPP does not predict measurement outcomes
- Instead, it uses past measurement outcomes to update belief distribution to plan paths

# **Experimental Results**

- Simulated Result:
  - Target and searchers move at 1 m/s
  - Randomized starting location for target
  - Simulate with museum floor plan of size 150m by 100m and office of size 100m by 50m
- Ranging Radio Measurements
  - Uses range sensors up to 30 m with errors 1-2 m
  - Robot acts as lost first responder, moving at .3 m/s

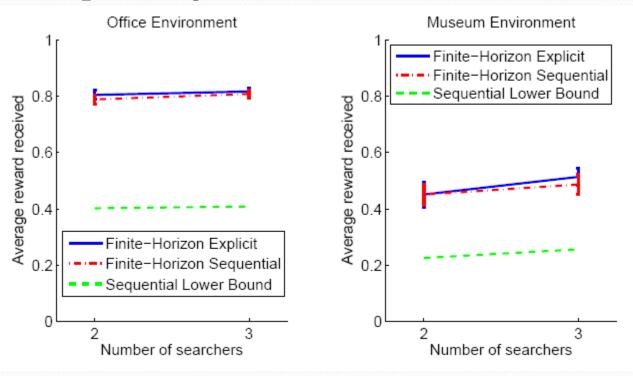
## Simulated Results

• Single robot finite horizon compared to optimal



## Simulated Results

 Sequential allocation compared to iteration through all search paths by all robots



# Ranging Radio Measurements

