Optimal Testing of Structured Knowledge

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Presented by Jon Napolitano

Testing Students

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  (Structured Knowledge)

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- Paper test or oral exam?
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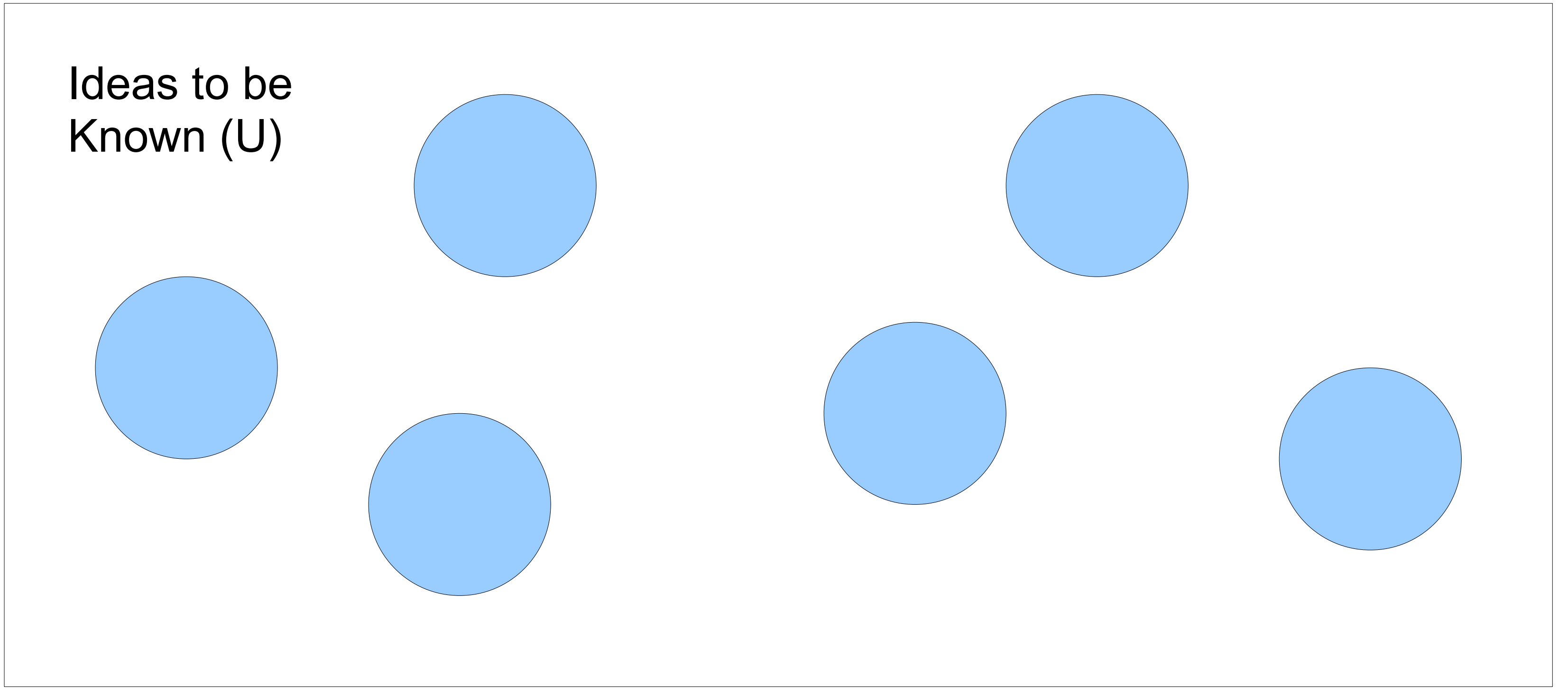
- One Professor, or multiple?
  (Single vs. Multi-agent learning)

Structured Knowledge



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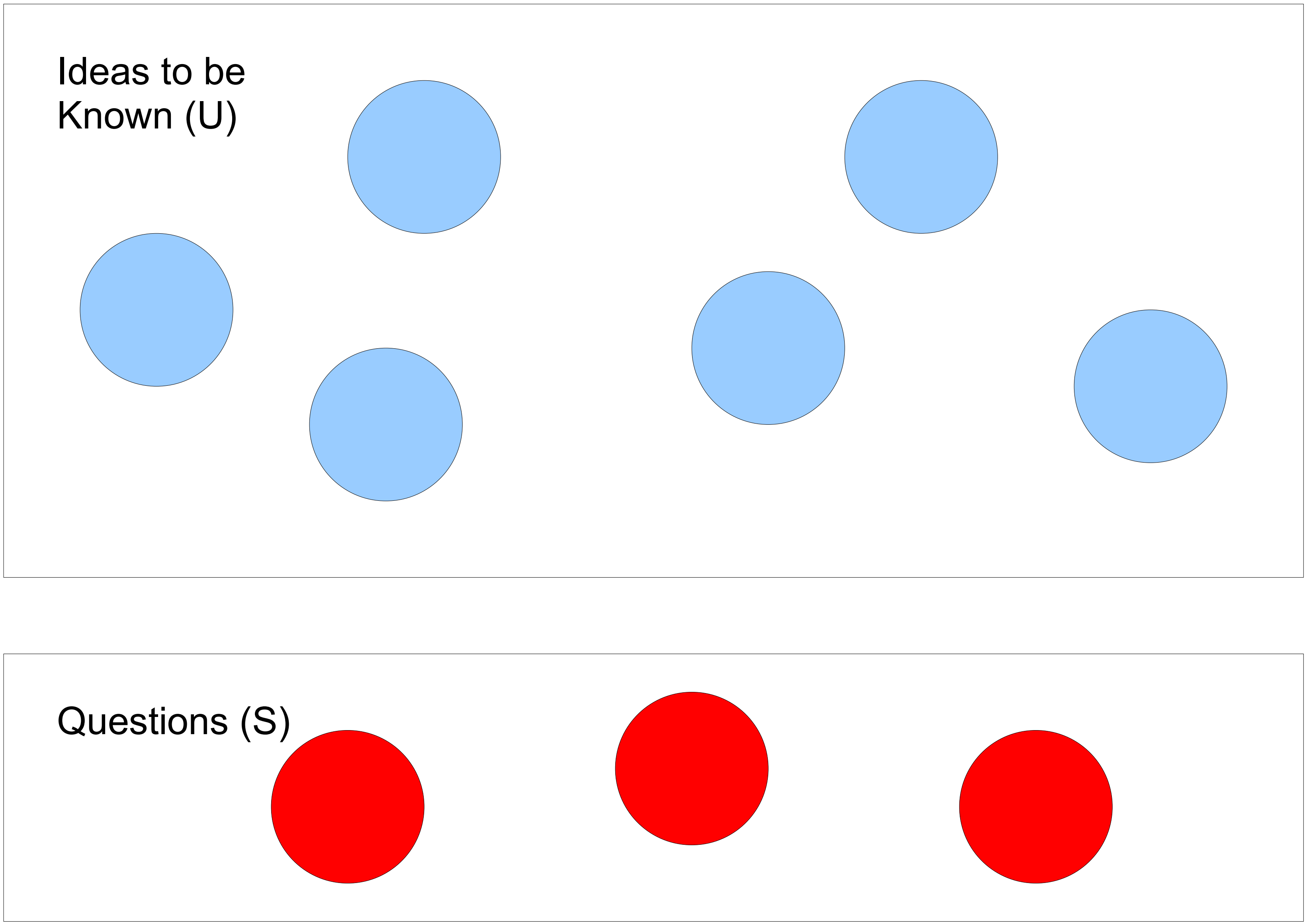

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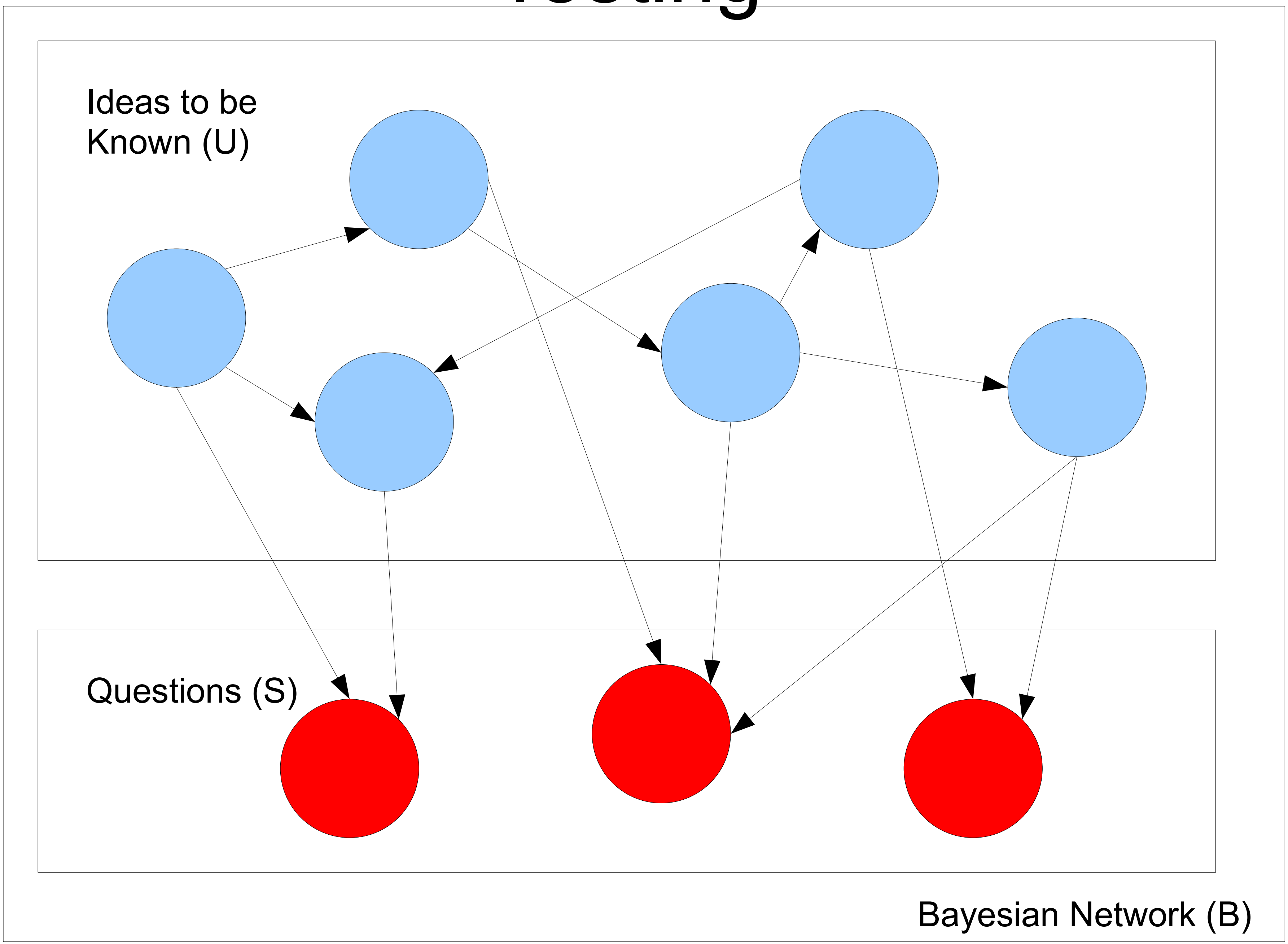

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- c : S -> **R**+
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- D = {0,1}
  (Decisions, i.e. Pass/Fail)
- A, the “design space”
  (Possible sets of questions within budget)

Utility Function

$$u = \begin{cases} 1 & \text{if } d=1 \text{ and } |\{x \text{ in } u: x=1\}| \geq z|U| \\ 1 & \text{if } d=0 \text{ and } |\{x \text{ in } u: x=1\}| \leq z|U| \\ 0 & \text{otherwise} \end{cases}$$

How do we decide?

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A: For this utility function, if
$P(\text{student knows his stuff} \mid \text{observations}) \geq .5$
then pass him.

How Hard is the Problem?

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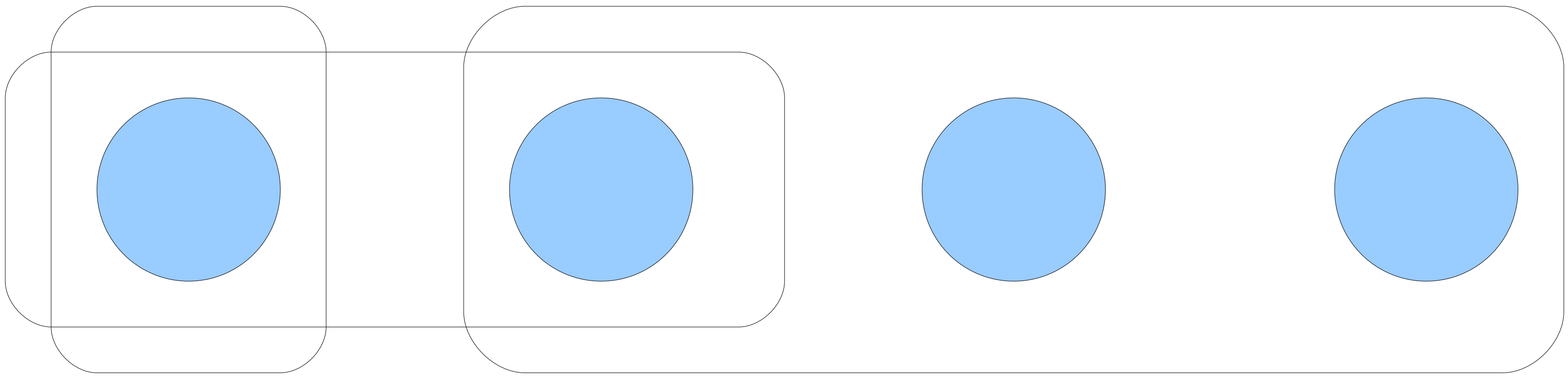
A: No.

Proof: Reduction from Set Cover



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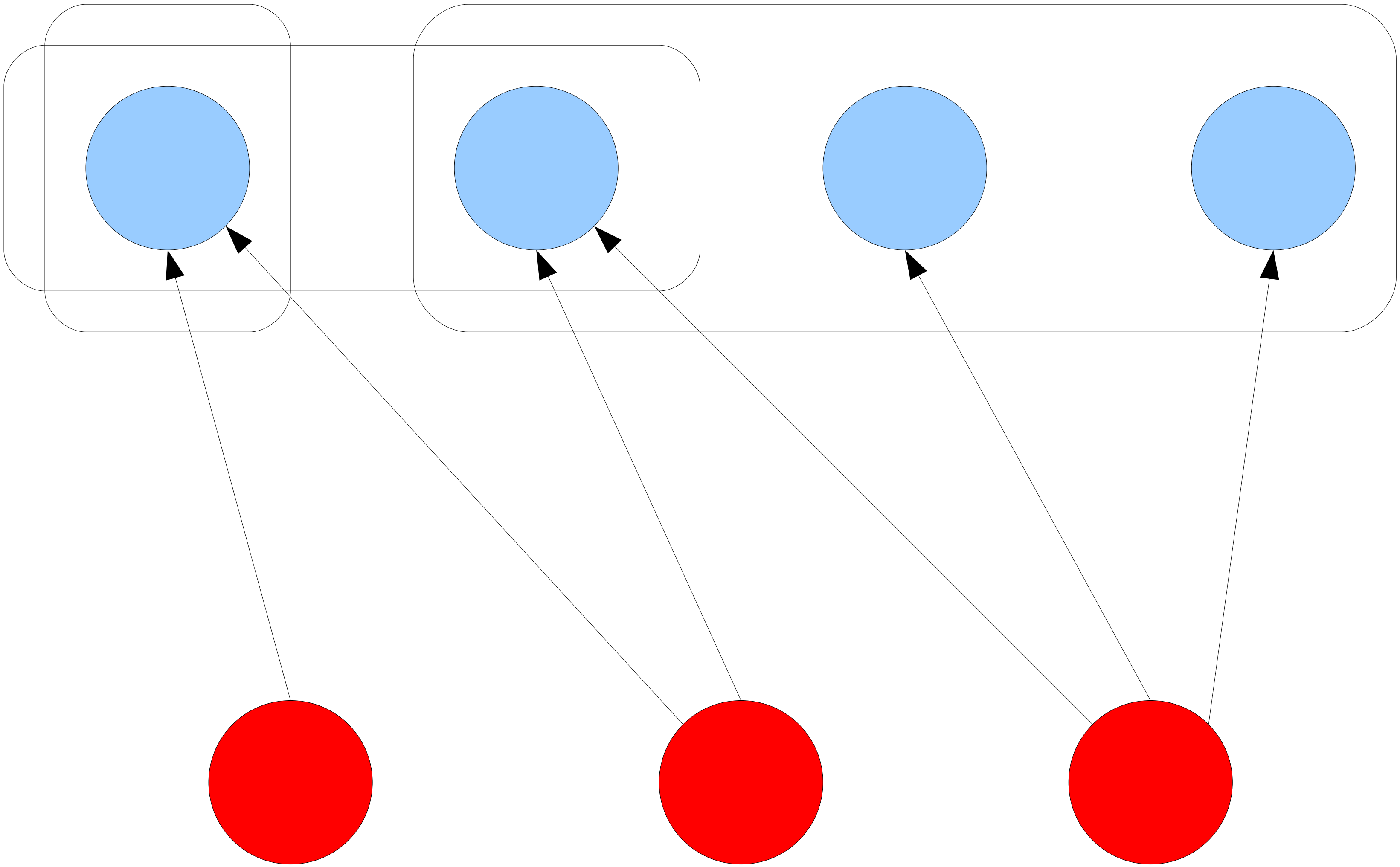

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- But we can if we restrict the graph a little bit!

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- Each parent in U is deterministically equal to its child in S.

Greedy Algorithm

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- This algorithm is optimal!

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- Given a budget b, improvement can be at least .5-(1/2^b).
- Still just as NP-hard.

Does Smooth Utility Help?

Define $f(d,u)$ such that
$f(Pass,u) \geq f(Fail,u)$ when $|\{x \text{ in } u: x=1\}| \geq z|U|$
and vice versa.

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$f(Pass,u) \geq f(Fail,u)$ when $|\{x \text{ in } u: x=1\}| \geq z|U|$
and vice versa.

No, it's still NP-hard by the same reduction,
and still relatively hard to approximate.

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- Grand committees view question-groups as a whole, and so can perceive the correctness better.
- Singleton sessions allow for more questions.
- So which is better?

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- For some networks, GCs do better than SSs with expected utilities of 1 vs. less than .5.
- For other networks, SSs do better than GCs with an expected utility of 1 vs. a utility equivalent to choosing based on the prior.
- Deciding which is the case is NP-hard!

Multiple Students

Fairness criterion:

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For every student, the value
H(D|A) (the entropy of the decision given the solution) should be identical.

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We can't ask every student a question, so the fair solution is to decide based on the prior, with expected utility of n/2

A Better Strategy

But if we choose instead to ask the question “what is 1+1?” of n-1 of the students, our expected utility becomes n-1/2, which is almost twice as good.

In Short

It's awfully hard to test well, and when you do it may not be fair.